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A new analytical approach for simplified thermal modelling of buildings: Self-Adjusting RC-network model

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ABSTRACT

A novel method for adjusting the parameters of a lumped parameter model for the transient thermal response of building constructions is presented. Previous analytical adjustment methods can be complex and inaccurate, while optimization algorithms, although more accurate, require prior simulation using a high-order reference model, and so provide no advantages to be integrated into simulation programs. This work presents a methodology for the analytical adjustment of a first-order model based on the hypothesis that the position of the capacitance varies in every time step in response to changes in the excitation value. By comparison with a reference model and using a wide range of constructions (420), the functional form of this dependence was determined in accordance with the value of time step and properties and thickness of the element. Using different typical constructions (41), the method was validated by comparison with the reference model in terms of surface heat fluxes, surface temperatures and indoor air temperature. The results showed an excellent agreement with the reference model for surface temperatures and indoor air temperature, and good agreement for surface heat fluxes. The method can be integrated into simulation programs with a low computational cost, sufficient accuracy, universality and adaptable time step.
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1. Introduction

The Finite Element Method and the Finite Difference Method are numerical methods that are extensively used to solve the partial differential equation that governs the conduction heat transfer through the construction elements of buildings, such as external walls, internal partitions, floors and roofs. Some building energy simulation programs, for example ESP-r [1,2], use the Finite Difference Method for calculating the transient thermal response of these elements. However, the computational cost of these methods is too high to be integrated into thermal simulation programs, so they are discarded by most developers. On the other hand, the Response Factor Method [3] or the Transfer Function Method [4] are the most used methods in building thermal simulation programs, such as EnergyPlus [5], TRNSYS [6] and others [7]. These methods are more efficient at calculating surface heat fluxes than the previous ones, because no knowledge is required of the temperature and heat flux distribution within the element. The Transfer Function Method [4] is based on the Response Factor Method [3], and allows the responses of the element to be obtained more efficiently. However, the solution become progressively more unstable as the simulation time step decreases. This is a problem in those cases requiring a short time step, such as those in which there is coupling with the HVAC systems and when the fabric dynamics can not be neglected. This problem can be partly resolved by using transfer function series obtained by means of the approximation of the heat equation by The Finite Difference Method [8,9]. However, using this solution involves a high computational cost, and there can still be difficulties with construction elements with a very high thermal capacity with time steps below 15 min. Another inherent drawback of the methods based on the transfer function is that the time step can not be changed during the simulation. For example, if the transfer function coefficients were obtained for a time step of one hour, the information about the surface temperatures and surface heat fluxes would be given equally every hour, with no possibility of obtaining this information at intermediate times. Some simulation programs, such as EnergyPlus [5], attempt this problem by using the 'Master history with interpolation' method [10].

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Nomenclature

A	Area (m^2)
C	Thermal capacity per unit area ($J/m^2\,K$)
C_p	Specific heat ($J/kg\,K$)
e	Thickness (m)
Fo	Fourier number
h	Convection heat transfer coefficient ($W/m^2\,K$)
k	Element constant ($1/^\circ C$)
$\dot{m}$	Mass flow rate (kg/s)
n	Number of layers of the element
$\dot{Q}$	Heat transfer rate (W)
$\dot{q}$	Heat flux (W/m^2)
R	Thermal resistance ($m^2\,K/W$)
T	Temperature ($^\circ C$)
t	Simulation time (s)
x	Distance (m)

Greek letters

α	Thermal diffusivity (m^2/s)
Δt	Simulation time step (s)
θ	Temperature increment ($^\circ C$)
λ	Thermal conductivity ($W/m\,K$)
ρ	Density (kg/m^3)
τ	Thermal time (s)

Subscripts and superscripts

$\circ$	Universal
*	Referred to capacitance node
1	Referred to surface 1 of the element
2	Referred to surface 2 of the element
a	Air
e	Element
eq	Equivalent
ext	External
hom	Homogeneous
i	Element surface index number
int	Internal
j	Element surface index number
l	layer index number of the element
o	Overall
s	Surface

In this method, the surface temperature and heat flux histories at intermediate instants of time are obtained by interpolation.

Unlike previous methods, lumped parameter methods for modelling the transient thermal response of construction elements of buildings offer simplicity and allow stable simulations in short time steps with a low computational cost, making them advantageous in comparison with detailed or high-order methods for their integration in building energy simulation programs or urban microclimate simulation programs.

Laret [11] and Lorenz and Masy [12] were the first authors to propose a simplified approach to modelling the thermal response of construction elements of buildings based on a first-order lumped parameter model composed of two resistances and one capacitance. Gouda et al. [13] proposed a second-order model in which each construction element is modelled using three resistances and two capacitances. Fraisse et al. [14] went further and developed a fourth-order model by adding to the second-order model (the latter referred to as a '3R2C' model) one capacitance on each surface of the element. The resulting model with three resistances and four capacitances was referred to as a '3R4C' model. Lumped parameter methods can also be applied for modelling the whole zone instead of individual construction elements [15–20]. In this approach, all the thermal capacities of the different construction elements that the zone is composed of (essentially, the higher thermal capacity elements, i.e. external walls, floors, etc.) are concentrated in a single equivalent capacitance, and another additional capacitance is added for the air of the zone (and internal furnishings). Thus, the result is a second-order model. Kämpf and Robinson [19] integrated an improved version of the model by Nielsen [18] in the CitySim [21] solver, a simulating program for urban planning inspired on its predecessor, SUNtool [22,23]. Another example of this approach is the hourly simplified method proposed by ISO 13790:2008 [24] for dynamic simulation of the evolution with time of the internal air and mean radiant temperatures of a building zone. This model, designed for simple and normative approach, is composed of five thermal resistances and one capacitance node representing the mass of the building zone (referred to as '5R1C' model). Although models based on this second approach offer great simplicity and very low computational cost, those based on the first approach provide more detailed and useful information for modelling (i.e. the surface temperatures of the individual elements forming the whole zone, which can be important for calculating the long wavelength radiation exchange), and maintain simplicity and a reduced computational cost. For this reason, this first approach is generally preferred to whole zone lumped parameter models.

However, the accuracy of these simplified models depends largely on the value of their characteristic parameters (resistances and capacitances), so they must be adjusted using a suitable method. Laret [11] proposed a simple analytical method for the division of the overall thermal resistance of the construction element between the two resistances of a first-order model by calculating a factor referred to as the 'accessibility factor'. However, the accuracy of this method is questionable in many situations, as observed by Gouda et al. [13]. Lorenz and Masy [12] and Fraisse et al. [14] proposed other analytical methods to obtain the value of the characteristic parameters of the model. However, these methods required complex mathematical models, so applying the model lost simplicity. In the case of Fraisse et al. [14], the step from a second-order model to a fourth-order one is somewhat arbitrary (the method involved transferring 5% from the two internal capacitances systematically to each of the surfaces of the element) and only one case of validation was presented. Ramallo-González et al. [25] presented an analytical method for generating a second-order lumped parameter model of multi-layered construction elements, which was built to be accurate in the range of the most significant frequencies of the inputs. On the other hand, Gouda et al. [13] used a method of optimization based on a single objective function search algorithm to determine the five free parameters of the second-order model by comparing the response with a high-order reference model. They compared first and second-order models and concluded that the latter were an improvement on the first as regards accuracy, and only required a small additional computational effort. However, their work was limited in several aspects, the main one being the fact that the characteristic parameters were adjusted using a unit step excitation on only one surface of the element. Later, Underwood [26] tried to overcome these limitations and proposed an improved method for adjusting the parameters of the second-order model proposed by Gouda et al. [13] based on a multiple objective function search algorithm that compared the response of the model with that obtained by a rigorous reference model. The end of the study included the value of the characteristic parameters of a wide group of typical construction elements. Crabb et al. [15] proposed a method based on that of Lorenz and Masy [12] for adjusting the resistances of a second-order lumped parameter model of the whole zone. However, the method provided unacceptable results for zones with elements with a very high thermal capacity. Tindale [16] tried to correct this by introducing a

third equivalent capacitance of the zone, but the parameterization involved a rather complex method.

In general, analytical adjustment methods can be complex and their accuracy is questionable in some cases, while methods based on optimization algorithms, although more accurate, require prior simulation using a detailed or high-order reference model, and so are not advantageous for their integration into buildings or urban microclimate thermal simulation programs (as the characteristic parameters of the model must be adjusted beforehand for each element).

Thus, it is necessary to find an analytical adjustment method (that does not require prior simulation using a reference model) that offers sufficient accuracy, maintains simplicity (therefore a low computational cost) and that can be adapted to changes in the value of the simulation time step.

The objective of this work is to address these requirements. A novel method named 'Self-Adjusting RC-network model' is proposed for the analytical adjustment of the characteristic parameters of a simplified lumped parameter model for the transient thermal response of building construction elements. To maintain simplicity, and hence speed at calculating, the simplified model is based on a first-order method with two resistances and one capacitance. The method is based on the hypothesis that the position of the capacitance within the element varies in every time step in response to changes in the excitation value. By comparison with a rigorous reference model and using an extensive group of construction elements (420), the functional form of this dependence was determined according to the value of the simulation time step and the thermal properties and thicknesses of the elements. Finally, using different typical construction elements (41), the method was validated at three levels depending of the kind of response to be compared with the reference model: surface heat fluxes, surface temperatures and indoor air temperature of a single zone.

2. Reference conduction model

As stated in Section 1, the parameters of the simplified lumped parameter model need to be adjusted. In the development of the adjustment methodology proposed in this study and also in the posterior validation, the response of the simplified model was compared with that obtained using a rigorous or high-order reference model.

The problem to solve is the transmission by conduction through a particular construction element. Assuming one-dimensional heat conduction, the partial differential equation (PDE) that governs the transient heat conduction through a slab of material is:

$$\frac{1}{\alpha}\cdot\frac{\partial T(t,x)}{\partial t}=\frac{\partial^2 T(t,x)}{\partial x^2} \tag{1}$$

In Eq. (1), $\alpha=\lambda/\left(\rho\cdot C_p\right)$ is the thermal diffusivity of the material. This PDE must be solved for each layer of material that the element is composed of.

The reference model is based on the well-known Finite Difference Method, a widely-used numerical method for solving the PDE that governs the transient one-dimensional heat conduction through the construction elements of buildings.

In this method, the element is divided into a finite number of slices whose state is represented by the temperature of one point (node) on its central plane, and the variation between adjoining nodes is assumed to be linear. The time evolution of the state of the element (the temperature of all the nodes it is formed of) is solved by dividing the time into a finite number of time steps and by applying energy balance to each node. This study used a backward or implicit difference scheme: the changes in temperatures from the current simulation time step to the next are calculated by making use of the temperatures at the next simulation time step. A detailed description of the method can be found in [27].

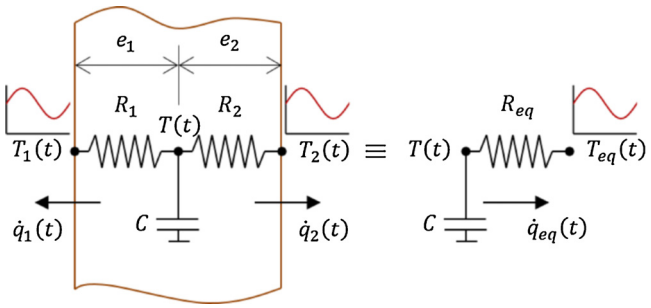


Fig. 1. Simplified first-order lumped parameter model (left: full RC-network; right: equivalent reduced RC-network).

3. First-order lumped parameter model

A simplified method to solve the problem of heat conduction through construction elements is using a lumped parameter approach. This approach can be understood as the approximation used to construct the rigorous reference model described in Section 2, but with a much smaller degree of discretization: the problem is approximated with a reduced number of thermal resistances and capacitances. For reasons of simplicity and speed at calculating, this study used a first-order lumped parameter model composed of two resistances and one capacitance (Fig. 1 (left side)). These thermal resistances and capacitances are the characteristic parameters of the model, so $R_1+R_2=R_o$ and $C=C_o$, where R_o and C_o are the overall surface-to-surface thermal resistance and the overall thermal capacity per unit area, respectively. Both can be calculated using the thickness and properties of each material:

$$R_o=\sum_{l=1}^{n}\frac{e_l}{\lambda_l} \tag{2}$$

$$C_o=\sum_{l=1}^{n}e_l\cdot\rho_l\cdot C_{pl} \tag{3}$$

The boundary conditions or input data of the model are the temperatures on surfaces 1 and 2 of the element (T_1 and T_2), and the responses or output data are the heat fluxes on both surfaces ($\dot{q}_1$ and $\dot{q}_2$), which can be calculated by Eqs. (4) and (5).

$$\dot{q}_1(t)=\frac{T(t)-T_1(t)}{R_1} \tag{4}$$

$$\dot{q}_2(t)=\frac{T(t)-T_2(t)}{R_2} \tag{5}$$

In Eqs. (4) and (5), T represents the temperature value of the capacitance node. Thus, to solve the first-order model of Fig. 1 (left side) involves obtaining the temperature value of the capacitance node in every simulation time step. Applying energy balance on the capacitance node, we can obtain the differential equation to calculate this temperature:

$$C\cdot\frac{dT(t)}{dt}=\frac{T_1(t)-T(t)}{R_1}+\frac{T_2(t)-T(t)}{R_2} \tag{6}$$

Eq. (6), can be solved using modular-graphical modelling tools, such as Modelica [28], or by equation-based tools, such as Modelica [28], EES [29] or MATLAB [30]. Alternatively, and to facilitate its implementation in simulation programs, the authors present their analytical solution, which has been obtained from the equivalent reduced RC-network represented in Fig. 1 (right side), where the

equivalent temperature and resistance (T_{eq} and R_{eq} respectively) are calculated as follows:

$$T_{eq}(t) = R_{eq} \cdot \left(\frac{T_1(t)}{R_1} + \frac{T_2(t)}{R_2} \right) \tag{7}$$

$$R_{eq} = \left(\frac{R_1 \cdot R_2}{R_1 + R_2} \right) \tag{8}$$

Applying energy balance again on the capacitance node of Fig. 1 (right side), we obtain:

$$C \cdot \frac{dT(t)}{dt} = \frac{T_{eq}(t) - T(t)}{R_{eq}} \tag{9}$$

The mathematical procedure for finding the temperature of the capacitance node of the previous differential equation can be consulted in [31], and its expression is given in Eq. (10). In this procedure, the authors considered a linear evolution of the excitation in every simulation time step, which is appropriate if we take into account that information of the excitation is usually given as discrete values separated by the time step.

$$T(t) \approx T_{eq}(t) + \frac{\tau}{\Delta t} \cdot (T_{eq}(t - \Delta t) - T_{eq}(t)) + \left[T(t - \Delta t) - \left[T_{eq}(t - \Delta t) + \frac{\tau}{\Delta t} \cdot (T_{eq}(t - \Delta t) - T_{eq}(t)) \right] \right] \cdot e^{-\frac{\Delta t}{\tau}} \tag{10}$$

In Eq. (10), $\tau = C \cdot R_{eq}$. As the previous equations show, the value of the response on each surface of the element depends on the value of the thermal resistances R_1 and R_2. Thus, for the simplified model to be completely parametrized, it is necessary to know how the overall surface-to-surface thermal resistance R_o is divided between the two resistances R_1 and R_2 that form the RC-network of the model. The accuracy of the responses largely depends on this division, so an adjustment method is required that has a good balance between simplicity and accuracy. In the following section, a novel analytical method is proposed for adjusting these parameters.

4. Proposal for the adjustment of the model parameters: self-adjusting method

Since the simplified model proposed by the authors is based on a first-order method with two resistances and one capacitance, the adjustment of the characteristic parameters involves finding the optimal division of the overall surface-to-surface resistance between the two resistances R_1 and R_2. This adjustment may depend on the kind of element (homogeneous or multi-layered element), its properties (high or low thermal capacity, and high or low thermal resistance), and the excitation. First, the case of a homogeneous element is considered. In this case, the thermal resistances R_1 and R_2 can be calculated as follows:

$$R_1 = \frac{e_1}{\lambda} \tag{11}$$

$$R_2 = R_o - R_1 \tag{12}$$

Where e_1 represents the distance between the surface 1 and the capacitance (Fig. 1 (left-side)). Consequently, the problem is limited to finding the optimal position of the capacitance inside the element, this being the only free parameter to determine. The value of the capacitance is equal to the overall thermal capacitance of the element.

The new methodology proposed is based on the following hypothesis: the position of the capacitance of the first-order lumped parameter model varies in every time step in response to changes in the excitation value. That is, the optimal division of the overall surface-to-surface resistance between the two resistances is not fixed, but rather changes over time in accordance with how the excitation does; this is a new approach with respect to what is proposed in previous adjustment methods.

Given that excitations are temperatures imposed on the two surfaces of the element, the problem can be divided into two sub-problems according to the surface onto which the excitation is imposed:

a) Sub-problem 1: temperature imposed onto surface 1 (T_1) and temperature equal to 0 °C on surface 2.
b) Sub-problem 2: temperature imposed onto surface 2 (T_2) and temperature equal to 0 °C on surface 1.

In turn, each one can be subdivided into another two sub-problems according to the surface of interest for calculating the heat flux response:

1) Sub-problem 1.1 (or sub-problem 2.1): Heat flux response on surface 1 ($\dot{q}_1$).
2) Sub-problem 1.2 (or sub-problem 2.2): Heat flux response on surface 2 ($\dot{q}_2$).

From here on, $\dot{q}_{i,j}$ represents the heat flux response on the surface i when temperature excitation is imposed on surface j. From the responses obtained for each previous sub-problem, it is possible to determine the heat flux on both surfaces. To do this, given that the PDE that governs the conduction heat transfer through the element is linear (Eq. (1)), it is enough to apply superposition on each surface (Fig. 2):

$$\dot{q}_1(t) = \dot{q}_{1,1}(t) + \dot{q}_{1,2}(t) \tag{13}$$

$$\dot{q}_2(t) = \dot{q}_{2,1}(t) + \dot{q}_{2,2}(t) \tag{14}$$

The methodology proposed is based on the variation, in every time step, of the position of the capacitance inside the element. In the present development, this position is measured with regard to an origin located on the surface onto which the excitation is imposed (e^*) (Fig. 2), and can be expressed in terms of the Fourier number (Fo^*):

$$Fo^*(t) = \frac{\alpha \cdot \Delta t}{e^*(t)^2} \tag{15}$$

The hypothesis suggests that this variation ΔFo^* must depend on the variation of $\Delta\theta$ with time by means of a given function f to be determined. Fig. 3 shows this approach for the case in which a temperature excitation is imposed on surface 1 and where the aim is to calculate the heat flux response on this surface (case a.1). In each time step, the variation of the position of the capacitance $\Delta Fo^*(t) = Fo^*(t) - Fo^*(t - \Delta t)$ depends on the variation of $\Delta\theta(t) = \theta(t) - \theta(t - \Delta t)$.

In principle, for any given element, function f may depend on the position of the capacitance itself, on the excitation and on the value of the simulation time step. This can be expressed analytically as:

$$\frac{dFo^*}{d\theta} = f\left(Fo^*, \theta, \Delta t\right) \tag{16}$$

To determine the functional form of the dependence between the variation of the position of the capacitance and the variation in the excitation value, that is, to find function f, a wide range of construction elements was used. Table 1 shows the different values of thermal properties and thickness used to generate the different construction elements. A set of 420 elements (single homogenous materials) was used, resulting from the combination of 5 different thickness values, 7 thermal conductivities, 4 densities and 3 specific heat values. For each element, a search for the function f was performed for random temperature imposed on one surface and heat

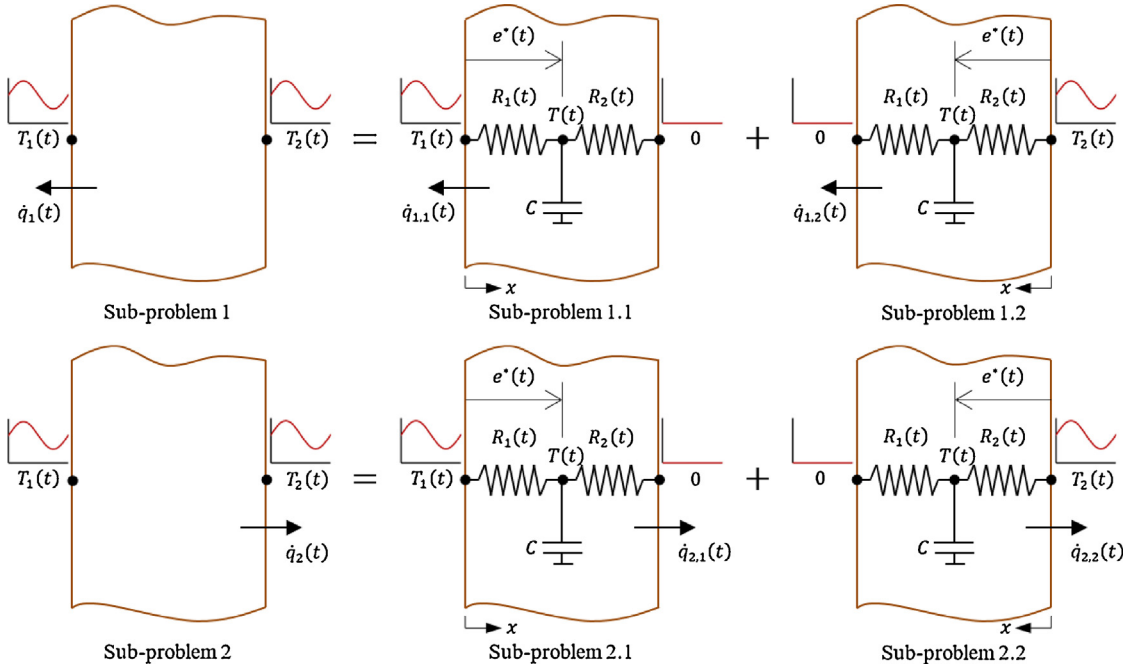


Fig. 2. Calculation of the surface heat fluxes $\dot{q}_1$ and $\dot{q}_2$ by applying superposition (top: superposition on surface 1; bottom: superposition on surface 2).

Table 1
Values of thermal properties and thicknesses used to generate the set of construction elements.

Thickness (mm)	Thermal conductivity (W/m K)	Density (kg/m^3)	Specific heat (J/kg K)
150	0.05	100	100
250	0.25	1000	1000
350	0.50	2000	2000
450	1.00	2500	–
500	1.50	–	–
–	2.00	–	–
–	2.50	–	–

flux response on this surface (case a.1 and case b.2), and for random temperature imposed on one surface and heat flux response on the opposite surface (case a.2 and b.1). In addition, three time steps were used (1 h, 15 min and 1 min).

An example is shown of the methodology followed to determine the function f for the case in which random temperature is imposed on one surface of the element (i) with the aim of calculating the heat flux on that surface (i) (case a.1 y case b.2). For each construction element, calculations were performed to find out the position of the capacitance which best fitted the response of the simplified model to the rigorous reference model at every time step. For this, a simple optimization algorithm was used that minimizes the deviation in every time step between the response obtained by the first-order simplified model and the reference model (in practice, the Excel method 'Generalized Reduced Gradient (GRG) Nonlinear' was used for the above [32]). The results showed that, for each element and value of the simulation time step, the variation of the position of the capacitance ΔFo^* with regard to the variation of $\Delta\theta$ is approximately equal to a constant of the element itself, that is, $f\left(Fo^*, \theta\right) \approx k_{ii}$. To achieve universality in the method, this constant must somehow be related with the value of the simulation time step and with the thermal properties and thickness of the specific element. Fig. 4 shows, in a log-log plot, the value of the constant k_{ii} obtained with the simple optimization algorithm in accordance with Fo_e for the three time steps used in this work ('Optimized' in the figure), where Fo_e is the Fourier number whose characteristic length is equal to the total thickness of the element ($Fo_e = \alpha \cdot \Delta t / e_o^2$). Using a multiple regression analysis, an expression was obtained that relates the constant of the element with the value of the simulation time step and the Fourier number Fo_e (Eq. (17)) ('Calculated' in the figure).

$$k_{ii} = k_{ii}^{\circ} \cdot \sqrt{\frac{\Delta t}{Fo_e}} \tag{17}$$

In Eq. (17), k_{ii}° is a universal constant whose value is $k_{ii}^{\circ} = 2.03 \times 10^{-5}\,{}^{\circ}\mathrm{C}^{-1}\,\mathrm{s}^{-1/2}$.

In practice, since the excitation data available are normally discrete values spaced by the time step (Δt), the derivative in Eq. (16) can be expressed in terms of increments:

$$\frac{Fo^*(t) - Fo^*(t-\Delta t)}{\theta(t) - \theta(t-\Delta t)} = k_{ii}^{\circ} \cdot \sqrt{\frac{\Delta t}{Fo_e}} \tag{18}$$

Once Fo^* has been obtained by Eq. (18), the position of the capacitance e^* can be calculated in every time step by using Eq. (15).

Similarly, the methodology followed to find function f of the element in cases a.1 and b.2 was repeated for cases a.2 and b.1; a temperature excitation was imposed on one surface (j) with the aim of calculating the heat flux on the opposite surface (i). In this case, the value of the constant is $k_{ij}^{\circ} = 0.0017 \times 10^{-5}\,{}^{\circ}\mathrm{C}^{-1}\,\mathrm{s}^{-1/2}$.

In the case of multi-layered construction elements, the same adjustment methodology was applied as for equivalent homogeneous elements whose thermal resistances and capacities are calculated as proposed below.

In cases where temperature is imposed on one surface with the aim of calculating the heat flux response on the same surface (cases a.1 and b.2), the methodology is applied to an equivalent homogeneous element whose thermal resistance is equal to the overall thermal resistance of the multi-layered element. The thermal capacity is equal to the sum of the thermal capacities of the different materials forming the multi-layered element, from the surface of interest to the insulation.

In the case of temperature imposed on one surface with the aim of calculating the heat flux response on the opposite surface

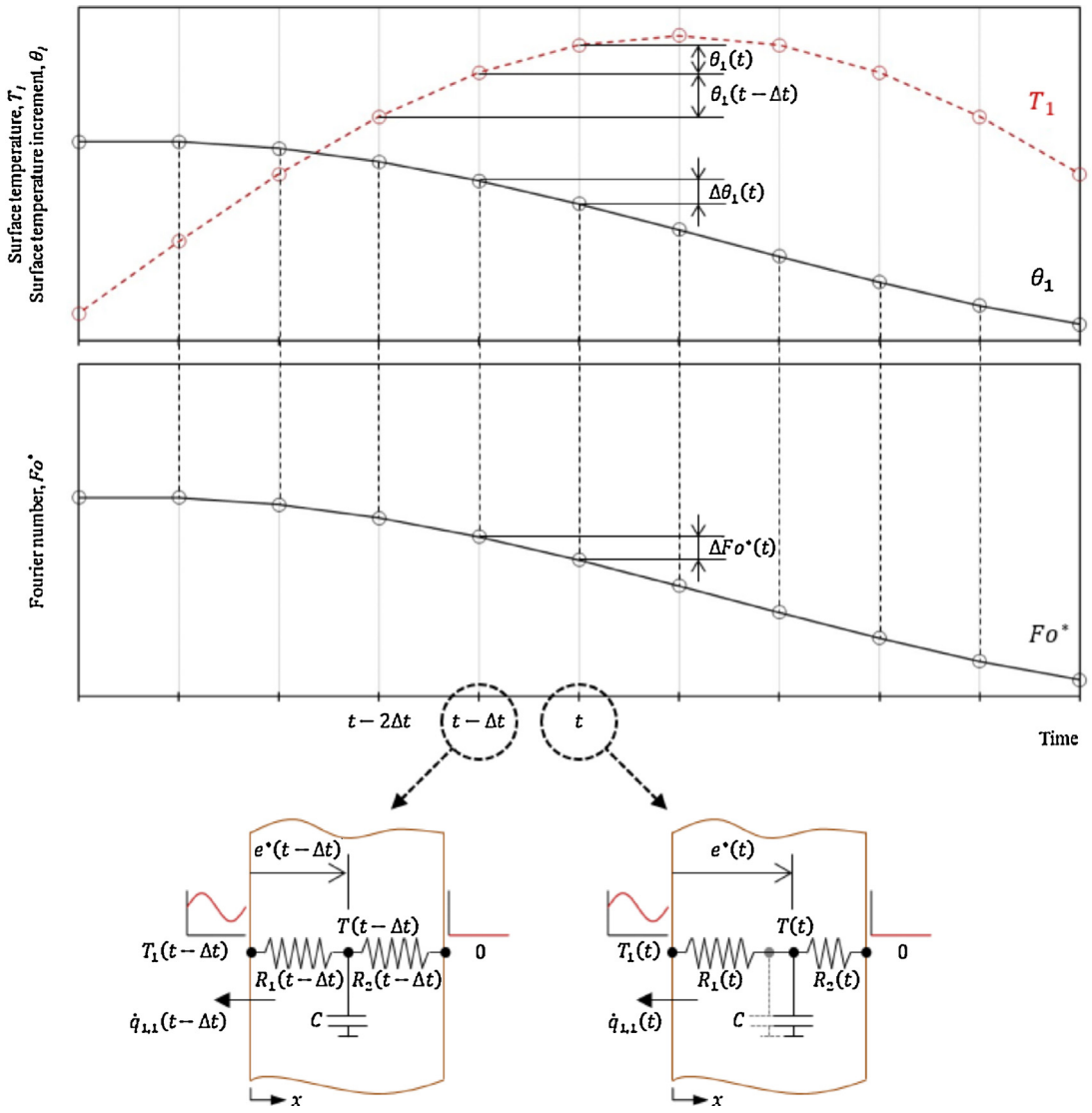


Fig. 3. Adjustment methodology proposed: variation of the position of the capacitance ΔFo^* in response to changes of $\Delta\theta$ (example for case a.1).

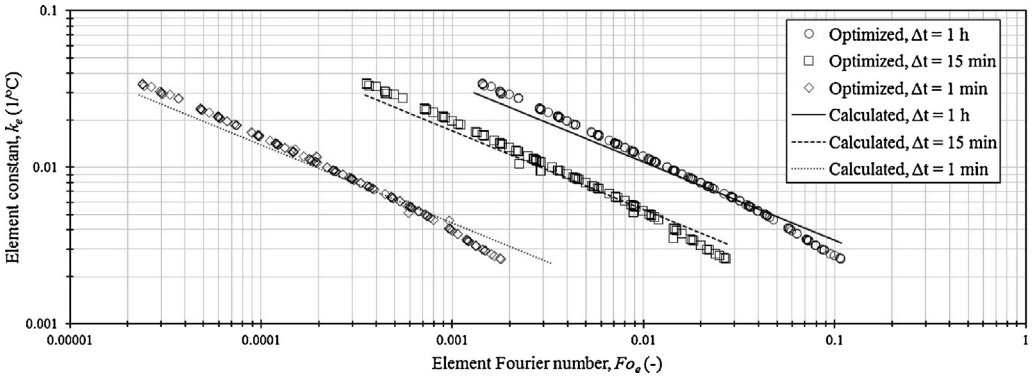


Fig. 4. Dependence of the element constant with the Fourier number Fo_e and the value of the time step.

(cases a.2 and b.1), the methodology is applied to an equivalent homogeneous element whose thermal resistance is equal to the overall thermal resistance of the multi-layered element and whose thermal capacity is equal to the sum of the thermal capacities of the different materials composing the multi-layered element, without taking into account the thermal capacity of the insulation.

Finally, if there is no insulation, the methodology is applied in both cases to an equivalent homogeneous element whose thermal resistance is equal to the overall thermal resistance of the multi-layered element and whose thermal capacity is equal to the overall thermal capacity of the multi-layered element.

The method proposed in this work makes it possible to adjust the position of the capacitance of a first-order simplified model in

every time step in accordance with changes in the excitation value, without needing a previous simulation with a rigorous reference model. For this reason, the authors have named the method 'Self-Adjusting RC-network model'.

5. Case studies for the model testing

To assess the accuracy of the proposed methodology for calculating the thermal response of construction elements of buildings, a three-level validation process was performed in accordance with the kind of output data to be compared with those obtained by the reference model:

a) Validation in terms of surface heat fluxes of the construction element.
b) Validation in terms of surface temperatures of the construction element.
c) Validation in terms of indoor air temperature of a square single zone formed with external walls.

In the three levels of validation, a set of typical construction elements were used including elements with a high and low thermal capacity and a high and low thermal resistance in order to assess the accuracy of the model for a wide range of values. These construction elements were selected from three different sources:

- Construction elements described in the ANSI/ASHRAE Standard 140-2004, Standard Method of Test for the Evaluation of Building Energy Analysis Computer Programs [33]. The construction elements in this standard are grouped into two categories: (a) lightweight elements, used for low mass basic tests (Cases 600 through 650) and (b) heavyweight elements, used for high mass basic tests (Cases 900 through 960). This study used the lightweight case exterior wall and roof, and the heavyweight case exterior wall (in this standard, the heavyweight case roof is the same as the lightweight case roof). The specification of the materials can be found in this standard (3 cases).
- Construction elements proposed by Underwood [26]. The external walls, internal partitions and roofs were selected from this group of typical construction elements. These are made of materials whose thermal properties are listed in CIBSE Guide, Book A [34] (37 cases).
- External wall construction described by Spitler [35]. Details related to the thickness and thermal properties of each of the layers can be found in [35] (1 case).

In the first validation level, the boundary conditions or input data for both the reference and simplified models were the temperatures on the external and internal surfaces of the element. The results or output data are the heat fluxes on both surfaces. For the external surface temperature, the typical meteorological year weather data of ANSI/ASHRAE Standard 140-2004 [33] ('DRYCOLD.TMY'), corresponding to the city of Denver, was used. This typical meteorological year is characterized by cold clear winters and hot dry summers, so it is appropriate for assessing the accuracy of the simplified model under excitation with abrupt temperature changes. The temperature of the internal surface varied sinusoidally with a mean value of 20 °C, 2 °C of amplitude and a period of 24 h.

In the second validation level, the boundary conditions for both models were the internal and external air temperatures. The results are the temperatures on the external and internal surfaces of the element. The internal and external air temperature values were the same as those used in the previous validation. The external and internal surface thermal resistances were set at 0.04 m^2 K/W and 0.13 m^2 K/W, respectively.

In the third validation level, the external walls selected for this study were formed into a simple square single zone with no interior partitions and no windows defined. The single zone was assumed to have dimensions of 10 m × 10 m and a height of 3 m. The five external walls were considered to be equal and the floor was treated as an adiabatic element. The input data in this validation were: the external air temperature, the internal convective heat transfer rate added directly to the zone air and the number of air changes per hour. The result is the temperature of the air of the single zone. The same values used in the first two validations were used for the external air temperature, while the internal air volume heat balance in Eq. (19) was used for the zone.

$$C_{a,int}\frac{dT_{a,int}}{dt} = \dot{Q}_{int} + \dot{m}_a \cdot C_{pa} \cdot (T_{a,ext} - T_{a,int}) + \sum_{e=1}^{5} A_s^e \cdot h_{int} \cdot (T_{s,int}^e - T_{a,int}) \quad (19)$$

In Eq. (19), $\dot{Q}_{int}$ is an internal convective heat transfer rate added directly to the zone air and $\dot{m}_a$ is a constant mass flow rate of external ventilation air that enters (and leaves) the single zone. As proposed by Underwood [26], an internal convective heat transfer rate of 1 kW was ramped in each day of the simulation between 07:00 h and 10:00 h and then ramped back down to zero between 18:00 h and 21:00 h after being held constant between these times. The mass flow rate of external ventilation air was calculated taking into account a constant rate of ventilation equivalent to 0.5 air changes per hour. The internal surface heat transfer coefficient h_{int} was calculated taking into account an internal surface thermal resistance of 0.13 m^2·K/W. The external surface thermal resistance was set at 0.04 m^2 K/W.

The output data corresponding to each previous validation level and to each typical construction element ware obtained separately using the reference model and by applying the methodology proposed in this study. These results were obtained over a complete simulation of one year. The results predicted with both models were compared and the Root-Mean-Square error (RMS error) was calculated in the corresponding terms according to the validation level. Table A.1 (Appendix A) shows the constants of each of the typical construction elements used for calculating the surface heat fluxes $\dot{q}_{1,1}$, $\dot{q}_{1,2}$, $\dot{q}_{2,2}$ and $\dot{q}_{2,1}$. From the knowledge of the previous surface heat fluxes, $\dot{q}_1$ and $\dot{q}_2$ were calculated by applying superposition. Each element constant was obtained from an equivalent homogeneous element whose thermal resistance and capacity were calculated in agreement with the methodology described in Section 4.

Table 2 shows the average RMS errors made by the simplified model with respect to the reference model, classified according to high or low overall thermal capacity, and high or low overall thermal resistance. In terms of surface heat fluxes (first validation level), the first-order model, which was adjusted by applying the methodology proposed in this study, showed good agreement with the reference model for the construction elements with a low thermal capacity. For these elements, the maximum and minimum average RMS error was 1.45 W/m^2 (3.22%) and 0.32 W/m^2 (0.60%), respectively. The level of agreement for elements with a high thermal capacity was adequate, though not as good as for those with a low thermal capacity. For these cases, the maximum and minimum average RMS error was 2.96 W/m^2 (5.53%) and 1.32 W/m^2 (2.94%), respectively. Regarding surface temperatures and indoor air temperature (second and third validation level), the model proposed presented a good fit with the reference model. The maximum and minimum average RMS error in terms of surface temperatures was 0.077 °C and 0.0001 °C, respectively. Regarding indoor air temperature, the maximum and minimum average RMS error was 0.2758 °C

Table 2
Average RMS errors in key variables between the reference and simplified models.

Level ofvalidation	Output data to be compared	High thermal capacity		Low thermal capacity	
		High thermal resistance	Low thermal resistance	High thermal resistance	Low thermal resistance
c)	Inside air temperature (°C)	0.0718	0.2758	0.0263	0.0238
b)	Inside surface temperature (°C)	0.0418	0.0473	0.0036	0.0231
	Outside surface temperature (°C)	0.0770	0.0079	0.0001	0.0018
a)	Inside surface heat flux (W/m^2)	1.3200	1.5900	0.3200	0.6400
	Outside surface heat flux (W/m^2)	2.9600	2.4400	1.4500	0.9400
b)	Monthly accumulated heat flux(kWh/m^2·month)	0.0144	0.1120	0.0064	0.0113

and 0.0238 °C, respectively. In addition, in the second validation level, the average RMS error of monthly accumulated heat flux in both surfaces of the element was calculated, finding a maximum and minimum average RMS error of 0.1120 kWh/m^2·month and 0.0064 kWh/m^2·month, respectively. Regarding computational effort, it was observed that the computer simulation time was reduced up to 34.4% and 63.4% for low thermal capacity and high thermal capacity constructions elements, respectively. A computer with an Intel(R) Core(TM) i3 2.30 GHz processor was used in the validation process.

As an example, Figs. 5–8 show the results obtained for the heavyweight case exterior wall of ANSI/ASHRAE Standard 140-2004 [33] by applying separately the reference model and the Self-Adjusting RC-network model proposed for each of the validation levels. The results are shown for four consecutive days. Table 3 shows the construction details of the element. Table 4 shows the value of the different element constants used for calculating the surface heat fluxes $\dot{q}_{1,1}$, $\dot{q}_{1,2}$, $\dot{q}_{2,2}$ and $\dot{q}_{2,1}$. Each constant was obtained from the Fourier number Fo_e of an equivalent homogeneous element whose thermal resistance $R_{hom,eq}$ and thermal capacity $C_{hom,eq}$ were determined as described in Section 4.

Fig. 5 shows a comparison of the results obtained with both models in terms of surface heat fluxes (first level of validation). The RMS error of the simplified model with respect to the reference model on the external surface was 0.17 W/m^2, while on the internal surface it was 0.71 W/m^2. Fig. 6 shows a comparison of the results obtained with both models in terms of surface temperatures (second level of validation). The RMS error made by the simplified model with respect to the reference model on the external and internal surfaces was 0.008 °C and 0.05 °C, respectively. Finally, Fig. 7 shows the air temperature of the single zone obtained with both models (third level of validation). The RMS error made by the simplified model with respect to the reference model was 0.12 °C.

For this element, the first-order model, adjusted using the methodology proposed, showed good agreement with the reference model at all the levels of validation. The adjustment methodology proposed enables the first-order simplified model to respond to abrupt changes in the different excitations and consequently to adjust with sufficient accuracy to the results obtained by the reference model. This behaviour is observed in particular on the external surface, where changes in excitation are more abrupt.

In addition, Fig. 8 shows the monthly results obtained for accumulated heat flux. The RMS error of the simplified model with respect to the reference model was 0.0024 kWh/m^2 month.

6. Conclusions

This work presents a new method for adjusting the characteristic parameters of a simplified lumped parameter model for the transient thermal response of building construction elements. Simplified models have the main advantage of having a lower computational cost compared with both rigorous and detailed methods. However, their accuracy depends heavily on the value of their characteristic parameters, so these must be adjusted using one of the available methods. Optimization methods that achieve a high level of accuracy require, for each specific element, a prior simulation using a rigorous reference model for the adjustment of the characteristic parameters. Thus, simplified models, whose characteristic parameters are adjusted using these optimization methods, provide no advantages in building or urban microclimate thermal simulation programs, and their use is limited to simulations that use elements whose characteristic parameters are already known.

This work presents a new methodology for the analytical adjustment of the characteristic parameters of a simplified model in which no prior simulation is required using a rigorous reference model. To maintain simplicity, and hence speed at calculating, the simplified model is based on a first-order method with two resistances and one capacitance. The method is based on the hypothesis that the position of the capacitance within the element varies in every time step in response to changes in the excitation value. The functional form of the variation of the position was obtained separately for a wide range of different construction elements (420) and different values of the time steps in the following cases: (a) temperature imposed on one surface and heat flux response on the same surface and (b) temperature imposed on one surface and heat flux response on the opposite surface (in both cases the temperature of the opposite surface remained equal to 0 °C). In both cases, using a simple optimization algorithm, calculations were performed to determine the position of the capacitance which best fitted the response of the simplified model to the rigorous reference model at every time step. In both cases, the results showed that, for a specific element and value of the time step, the variation of the position of the capacitance (expressed in terms of Fourier number) with regard to the variation in the excitation value is equal to a constant of the element itself, and different in the case of (a) and (b). Then, so that the method is universally applicable, the value of these constants was found in accordance with the value of the time step and the properties and thickness of the specific element. Applying superposition on each surface, that is, adding the responses obtained separately for (a) and for (b), the responses can be obtained for a particular element. Therefore, the method proposed in this study makes it possible to adjust the characteristic parameters of a first-order simplified model in every time step in accordance with changes in the excitation value, without needing a previous simulation with a rigorous reference model. For this reason, the authors have named the method 'Self-Adjusting RC-network model'.

To assess the accuracy of the simplified model, adjusted using the methodology proposed here, the results obtained for a wide range of typical construction elements (41) were compared with the rigorous reference model based on an implicit Finite Difference Method. This comparison was performed at three levels, in accordance with the kinds of output data. In the first level of comparison, surface heat flux results were compared, the second level compared surface temperatures and, finally, the third level contrasted the air temperature of a single zone formed of several of

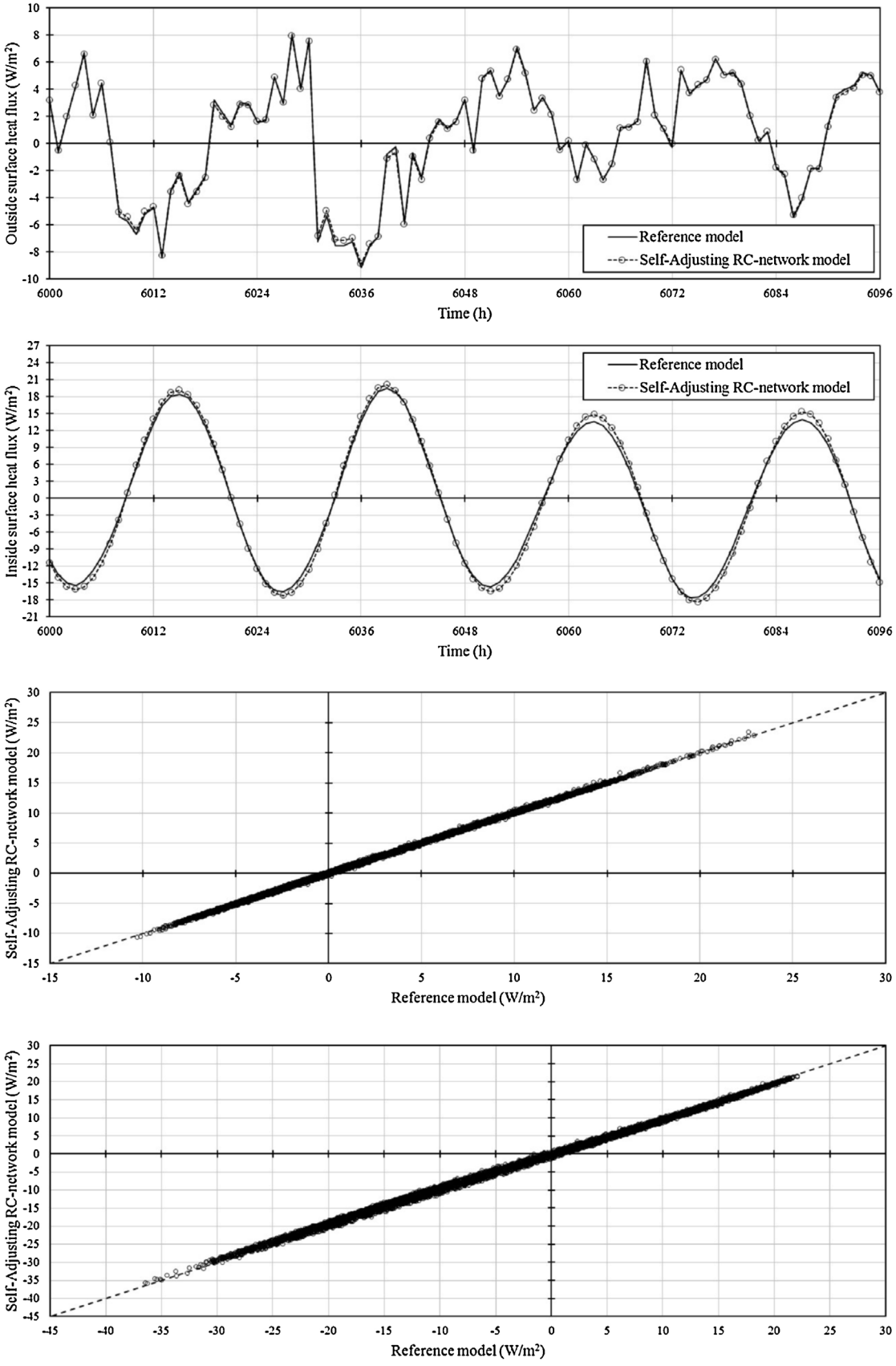


Fig. 5. Outside and inside surface heat flux (heavyweight case exterior wall of [33]).

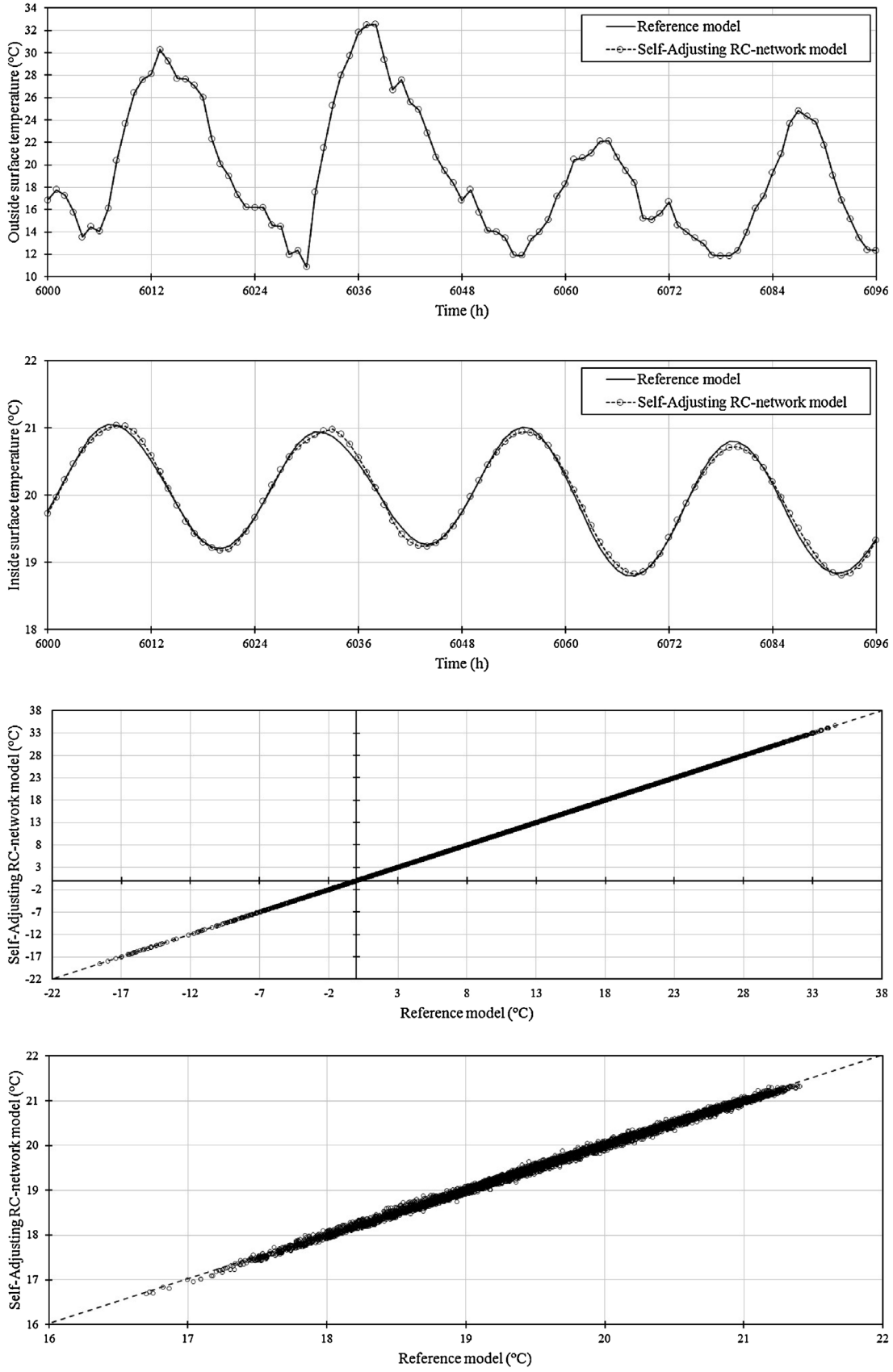


Fig. 6. Outside and inside surface temperature (heavyweight case exterior wall of [33]).

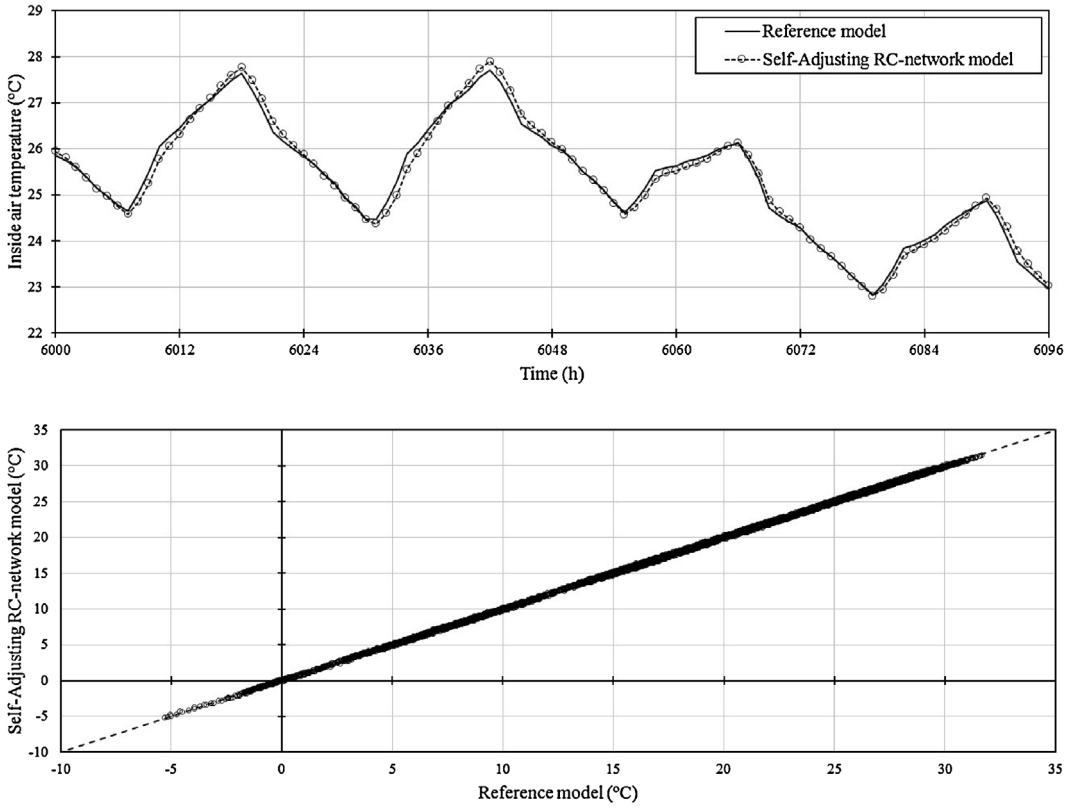


Fig. 7. Air temperature of the single zone (heavyweight case exterior wall of [33]).

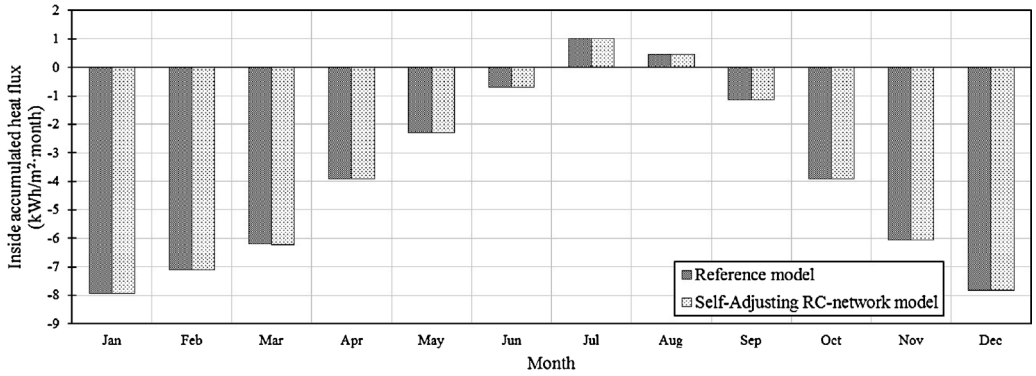


Fig. 8. Inside monthly accumulated heat flux (heavyweight case exterior wall of [33]).

Table 3
Heavyweight case exterior wall of ANSI/ASHRAE Standard 140-2004 [33] construction details.

Material (from outside to inside)		Thickness (mm)	Thermal conductivity (W/m K)	Density (kg/m^3)	Specific heat (J/kg K)
1	Wood Siding	9.0	0.140	530	900
2	Foam Insulation	61.5	0.040	10	1400
3	Concrete Block	100.0	0.510	1400	1000

Table 4
Heavyweight case exterior wall of ANSI/ASHRAE Standard 140-2004 [33] constants.

Problems to be solved		$R_{hom,eq}$ (m^2 K/W)	$C_{hom,eq}$ (J/m^2 K)	Fo_e (−)	k_e (1/°C)
$\dot{q}_1$	$\dot{q}_{1,1}$	1.7979	4293	0.4664	1.7834E-03
	$\dot{q}_{1,2}$	1.7979	144293	0.0139	8.6600E-06
$\dot{q}_2$	$\dot{q}_{2,2}$	1.7979	140000	0.0143	1.0184E-02
	$\dot{q}_{2,1}$	1.7979	144293	0.0139	8.6600E-06

the construction elements selected in this work. The constants of the method corresponding to each construction element have been presented. For the three levels of validation, the average RMS errors have been presented classified according to elements of high or low thermal capacity, and high or low thermal resistance. The results of the tests showed that the simplified model has excellent agreement with the reference model in terms of surface temperatures and indoor air temperature for elements of both high and low thermal capacity. In terms of surface heat fluxes, the first-order model shows good agreement with the reference model for construction elements with a low thermal capacity. The level of agreement for elements with a high thermal capacity is adequate, though not as good as for low thermal capacity. A reduction in computational time of up to 34.4% and 63.4% was observed for low thermal capacity and high thermal capacity constructions elements, respectively.

The main advantages of the method are:

1) Simplicity and a low computational cost. This advantage is inherent to first-order lumped parameter models.
2) Accuracy. As the new methodology is based on the movement of the position of the capacitance, the first-order simplified model can respond to abrupt changes in excitation and consequently can adjust better than current RC-network methods to the response obtained by the reference model, improving the accuracy.
3) Universality. The universal constants were obtained, thus eliminating the need for a previous simulation with a rigorous reference model to perform the adjustment of the characteristic parameters.
4) Possibility to change the value of the time step during the simulation. As an inherent characteristic of RC-network models, unlike models based on the conduction Transfer Function Method, it is possible to change the value of time step during a simulation.
5) Applicability for any kind of excitation. The methodology presented in this study is based on the variation in the excitation value in every time step, so the simplified model can be applied for any kind of excitation.

Therefore, this method can be integrated into building thermal simulation programs with a low computational cost, sufficient accuracy, universality and with an adaptable simulation time step.

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Appendix A. Simplified model constants for the validation cases

Table A.1
Self-Adjusting RC-network model constants of the typical construction element set.*

Construction element		R_o (m^2·K/W)	C_o (J/m^2·K)	k_{11} (1/°C)	k_{12} (1/°C)	k_{22} (1/°C)	k_{21} (1/°C)
Construction elements of ANSI/ASHRAE Standard 140-2004 [33]:							
1	Lightweight Case: Exterior Wall	1.7893	14534.3	1.7821E-03	2.1816E-06	2.6616E-03	1.4607E-06
2	Heavyweight Case: Exterior Wall	1.7979	145154.0	1.7864E-03	8.3616E-06	1.0201E-02	8.3616E-06
3	Lightweight Case: Roof	2.9932	18169.9	3.3490E-03	2.5758E-06	3.1425E-03	2.7451E-06
Construction elements proposed by Underwood [26] (from outside to inside):							
1	PB13/MFS200/SW20	5.8035	38650.0	5.8116E-03	4.0895E-06	4.9891E-03	4.7636E-06
2	PB13/MFS300/SW20	8.5813	45750.0	7.0668E-03	4.9728E-06	6.0668E-03	5.7925E-06
3	PB13/MFS200/MC05	5.6369	40702.0	6.1308E-03	4.0303E-06	4.9170E-03	5.0253E-06
4	PB13/MFS300/MC05	8.4147	47802.0	7.4906E-03	4.9243E-06	6.0076E-03	6.1399E-06
5	PB13/CBL105/MFS200/B105	6.1459	268006.0	1.4970E-02	1.6814E-05	2.0513E-02	1.2271E-05
6	PB13/CBL105/MFS300/B105	8.9237	275106.0	1.8039E-02	2.0261E-05	2.4718E-02	1.4786E-05
7	PB13/CBH105/MFS200/B105	6.0670	310342.0	1.4874E-02	1.8720E-05	2.2838E-02	1.2192E-05
8	PB13/CBH200/MFS200/B105	6.1395	489094.0	1.4963E-02	2.5680E-05	3.1330E-02	1.2264E-05
9	PB13/CBH105/MFS300/B105	8.8447	317442.0	1.7959E-02	2.2603E-05	2.7575E-02	1.4721E-05
10	PB13/CBH200/MFS300/B105	8.9173	496194.0	1.8033E-02	3.0949E-05	3.7758E-02	1.4781E-05
11	CBL105/MFS200/B105	6.0646	257632.0	1.4871E-02	1.6171E-05	1.9729E-02	1.2189E-05
12	CBH200/MFS300/B105	8.8360	485820.0	1.7950E-02	3.0392E-05	3.7078E-02	1.4713E-05
13	CBL105/MFS300/MC05	8.4925	192660.0	7.5252E-03	1.9136E-05	2.3346E-02	6.1682E-06
14	CBH105/MFS300/MC05	8.4136	234996.0	7.4901E-03	2.1488E-05	2.6215E-02	6.1395E-06
15	CBH200/MFS300/MC05	8.4861	413748.0	7.5223E-03	2.9784E-05	3.6336E-02	6.1659E-06
16	PB13/MFS250/RT10	7.0376	43324.0	6.6503E-03	4.5033E-06	5.4941E-03	5.4511E-06
17	PB13/MFS300/SW20/RT10	8.5932	60950.0	1.0199E-02	4.9762E-06	6.0710E-03	8.3595E-06
18	PB13/MFS250/WS50/AS10	7.5344	72696.0	1.1783E-02	4.6596E-06	5.6847E-03	9.6584E-06
19	PB13/MFS250/ACS100/AS10	7.6594	89696.0	1.3964E-02	4.6981E-06	5.7316E-03	1.1446E-05
20	PB13/MFS250/CH200/AS10/SC10	7.1987	401696.0	3.3344E-02	4.5546E-06	5.5566E-03	2.7331E-05
21	PB13/MFS300/SW20/AS10	8.5899	65322.0	1.0932E-02	4.9753E-06	6.0698E-03	8.9603E-06
22	SW20/MFS300/MC05	8.5001	51504.0	7.5285E-03	5.7650E-06	7.0333E-03	6.1709E-06
23	PB13/MFS300/MC05	8.4147	47802.0	7.4906E-03	4.9243E-06	6.0076E-03	6.1399E-06
24	PB13/PB13	0.1625	20748.0	1.1807E-03	9.6775E-07	1.1807E-03	9.6775E-07
25	PB13/MFS75/PB13	2.2458	26073.0	3.1036E-03	2.5440E-06	3.1036E-03	2.5440E-06
26	PB13/MFS100/PB13	2.9403	27848.0	3.5512E-03	2.9108E-06	3.5512E-03	2.9108E-06
27	B105	0.3500	88200.0	3.5725E-03	2.9283E-06	3.5725E-03	2.9283E-06
28	B220	0.7333	184800.0	7.4853E-03	6.1355E-06	7.4853E-03	6.1355E-06
29	PB13/B105/PB13	0.5125	108948.0	4.8047E-03	3.9383E-06	4.8047E-03	3.9383E-06
30	PB13/B220/PB13	0.8958	205548.0	8.7253E-03	7.1519E-06	8.7253E-03	7.1519E-06
31	P13/B105	0.4065	96062.4	4.0182E-03	3.2936E-06	4.0182E-03	3.2936E-06
32	P13/B105/P13	0.4630	103924.8	4.4605E-03	3.6561E-06	4.4605E-03	3.6561E-06
33	P13/B220	0.7899	192662.4	7.9320E-03	6.5016E-06	7.9320E-03	6.5016E-06
34	P13/B220/P13	0.8464	200524.8	8.3767E-03	6.8662E-06	8.3767E-03	6.8662E-06
35	CBL200	0.3030	295680.0	6.0864E-03	4.9889E-06	6.0864E-03	4.9889E-06

Table A.1 (*Continued*)

Construction element		R_o (m^2·K/W)	C_o (J/m^2·K)	k_{11} (1/°C)	k_{12} (1/°C)	k_{22} (1/°C)	k_{21} (1/°C)
36	P13/CBL200	0.3596	303542.4	6.7174E-03	5.5060E-06	6.7174E-03	5.5060E-06
37	P13/CBL200/P13	0.4161	311404.8	7.3191E-03	5.9992E-06	7.3191E-03	5.9992E-06
External wall construction described by Spitler [35] (from outside to inside):							
1	PB13/SCB100/INS50/FB100	1.5075	358116.4	9.7845E-03	9.1675E-06	1.1184E-02	8.0201E-06

* k_{11}, k_{12}, k_{22} and k_{21} represent the element constants used to calculate $\dot{q}_{1,1}$, $\dot{q}_{1,2}$., $\dot{q}_{2,2}$ and $\dot{q}_{2,1}$ respectively.

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