The disjunction introduction rule: Syntactic and semantics considerations

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ABSTRACT: The mental logic theory claims that the disjunction introduction rule, which is a valid rule in classical logic, is not a rule naturally used by the human mind. However, a study carried out by Orenes and Johnson-Laird suggests that this theory is wrong because there are cases in which the disjunction introduction rule appears to be applied by individuals. Orenes and Johnson-Laird explain their results based on the mental models theory. Nevertheless, in this paper, I try to show that, if it is assumed that people recover, by means of a semantic analysis of possibilities, the real logical forms of propositions before using formal rules, the mental logic theory can also explain the results achieved by Orenes and Johnson-Laird. Likewise, I include some comments on the possibility that both theories are complementary and related.

KEY WORDS: disjunction introduction rule; logical forms; mental models; semantics; syntax.

SUMMARY: 1. Introduction. 2. A paradox related to disjunction: The disjunction introduction rule. 3. The disjunction introduction rule and modulation. 4. The logical forms and the modulated versions. 5. Conclusions.

Fecha de Recepción 13/08/2014
Fecha de Revisión 13/09/2015
Fecha de Aceptación 16/10/2015
Fecha de Publicación 01/12/2015
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1. INTRODUCTION

An interesting debate has characterized cognitive science over recent decades. That debate is that related to the relations between logic and human mind. It can be said that there are different theoretical positions in this respect. However, in this paper, I will only consider two of them: the mental models theory (e.g., Byrne & Johnson-Laird, 2009; Johnson-Laird, 1983, 2001, 2006, 2010, 2012; Johnson-Laird & Byrne, 2002; Johnson-Laird, Byrne, & Girotto, 2009; Oakhill & Garnham, 1996; Orenes & Johnson-Laird, 2012) and the mental logic theory (e.g., Braine & O’Brien, 1998a; O’Brien, 2009). The reason why I will only focus on these two theories is that, as it has been shown by López Astorga (2013), many cognitive phenomena can be explained both by the mental models theory and by an approach based on the formal rules of logic. In this way, it seems appropriate to look for facts or experimental results that are consistent with one of these two frameworks and incompatible with the other one.

As it is well known, the mental models theory states that human reasoning works by means of the analysis of the possibilities corresponding to propositions. Thus, individuals tend to only assume the possibilities, or models, to which the premises refer and that are coherent with the conclusions. A good example can be its account of the Modus Ponens logical rule \((p \rightarrow q, p \vdash q)\). According to this theory, a conditional sentence such as \(p \rightarrow q\) has three models:

\[
\begin{align*}
(1) & \quad p \land q \\
(2) & \quad \neg p \land q \\
(3) & \quad \neg p \land \neg q
\end{align*}
\]

Apart from the fact that only (1) is an explicit model (following the mental models theory, (2) and (3) are implicit models that can only be recovered by means of certain cognitive effort), if we know that \(p\) is true, the only possible model is (1), since \(p\) is false - \(\neg p\) is true – in the other two models. This means that there is no possible scenario in which \(p\) is true and \(q\) is false.

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1 This paper is a result of the Project N. I003011, “Algoritmos adaptativos e inferencias lógicas con enunciados condicionales”, Supported by the Directorate for Research of Talca University (Dirección de Investigación de la Universidad de Talca), Chile. The author and lead researcher of this Project would like to thank the mentioned institutions for their help in funding this paper.
On the other hand, the mental logic theory proposes that human reasoning is led by formal rules. Nevertheless, given that this theory is not always correctly understood and it is often misinterpreted, it is important to highlight that, according to it, human beings do not necessarily reason by means of the rules of the classic propositional calculus. This theory only tries to describe the rules that people use in a natural way, and, although many of those rules are rules of the classic logic, not all the rules of this logic are admitted by the mental logic theory. For example, an especially relevant datum for this paper is that, according to this approach, people do not usually use the disjunction introduction rule \( (p \vdash p \lor q) \) (Braine, Reiser, & Rumain, 1998).

The basic problem is that both theories have a great descriptive and predictive potential and that, as said, it appears to be necessary to find decisive data that allow us to choose one of them. Orenes and Johnson-Laird’s (2012) paper seems to give such data. The results of an experiment carried out by them show that, although it is true that, in many cases, people do not apply the disjunction introduction rule, there are also cases in which they do apply that rule. This circumstance is a challenge for the mental logic theory, since the mental models theory can explain it and, at least in principle, it appears that the mental logic theory cannot do it. Certainly, it is not hard for the mental logic theory to explain why individuals do not tend to use that rule. What is difficult for it is to explain why they do use that rule in some cases.

Nonetheless, in this paper, I will try to prove that Orenes and Johnson-Laird’s (2012) findings cannot be considered to be decisive facts in support of the mental models theory and against the mental logic theory. In my view, if it is assumed that people, before making inferences, process information in order to detect the real logical forms of propositions, that is, that there is a phase prior to reasoning in which individuals interpret information, it can be said that the mental logic theory can also explain the results obtained by Orenes and Johnson-Laird (2012).

Maybe both theories are not incompatible, and it is very possible that each of them focuses on different intellectual activities. In any case, all of these issues will be addressed in the next pages. I will begin describing the problems of the disjunction introduction rule and the account of them given by the mental models theory. Then I will comment the experiment carried out by Orenes and Johnson-Laird (2012), the results achieved by them, and the interpretation of such results that they offer. Finally, I will explain how, if the real logical forms of the propositions used in that experiment are identified, it can be argued that the mental logic theory is also consistent with those results.
2. A PARADOX RELATED TO DISJUNCTION: THE DISJUNCTION INTRODUCTION RULE

Really, the main problem of this paper does not refer to a paradox. That is the term used by Orenes and Johnson-Laird (2012), who, indeed, are aware that it is not a true paradox, with the intention of indicating that there is a difficulty related to human reasoning that needs to be explained. That difficulty is as follows.

Faced with a proposition, people tend to reject a conclusion derived from it in which that proposition is linked to another by means of a disjunction (that is, people tend to reject a disjunctive conclusion in which the initial proposition is only the first disjunction). In other words, the problem is that, as mentioned above, individuals do not often accept a valid logical rule, the disjunction introduction rule. Orenes and Johnson-Laird (2012) offer a thematic example of this controversial inference, which is considered by them as a ‘paradoxical version’ of it. That example is this one:

“Viv is here. Therefore, Pat is here or Viv is here, or both” (Orenes & Johnson-Laird, 2012: 362).

Based on the mental models theory, Orenes and Johnson-Laird (2012) explain this fact. The models corresponding to the conclusion, if the disjunction is interpreted as inclusive, are the following:

(A) Pat is here – Viv is here
(B) Pat is here – Viv is not here
(C) Pat is not here – Viv is here

These three models refer to the situations in which the conclusion is true, but the key is the model (B). (B) is the key because it refers to a situation in which the premise, ‘Viv is here’, is false. Therefore, following the mental models theory, it is not surprising that individuals tend to reject this inference. The cause is that the conclusion allows a semantic possibility that is incompatible with the premise.

Obviously, this fact could be interpreted as evidence that the mental models theory holds, since it appears to show that people only reason considering semantic models, and not formal or syntactic rules. However, this problem does not really affect theories such as the mental logic theory. As indicated, this last theory does not state that human mind works resorting to all the formal rules of classical standard logic. Individuals do not often use some of such rules, including precisely the disjunction introduction rule.
The real problem for the mental logic theory seems to be that Orenes and Johnson-Laird (2012) carry out an experiment with versions (called ‘modulated versions’ by them) of this inference that are accepted as valid by most of their participants. As it will be commented in the next section, the mental models theory can easily explain such results. The question is whether or not the mental logic theory can do so as well.

3. THE DISJUNCTION INTRODUCTION RULE AND MODULATION

According to the mental models theory, sentences, and hence inferences, can be modulated in order to block or to facilitate models. Modulation can be related to different factors. Nevertheless, as far as the issue of this paper is concerned, it can be said that there is a modulation mechanism that can transform disjunctive sentences in such a way that people consider as valid inferences such as that of Viv and Pat described in the previous section.

Their idea is to create inferences with that same structure but, at the same time, with a special characteristic: in the disjunctive conclusion, the first disjunction cannot be false if the second one is true. Their example is as follows:

“Lucia wore Jewelry.

Therefore, Lucia wore the bracelet or she wore jewelry” (Orenes & Johnson-Laird, 2012: 363).

As it can be noted, the special feature of this inference is that it does not refer, as the previous one, to three models, but only to two:

(A) Lucia wore the bracelet – Lucia wore jewelry
(C) Lucia did not wear the bracelet – Lucia wore jewelry

Evidently, (B) is not possible now because a scenario in which Lucia wears the bracelet and she does not wear jewelry cannot be thought. A bracelet is jewelry and, therefore, if somebody wears a bracelet, that same person wears jewelry.

Thus, in the modulated versions, the conclusion is not linked to models inconsistent with the premise, and that circumstance causes people to accept these versions. This is the account of the mental models theory, and Orenes and Johnson-Laird check it by means of their experiment. Certainly, they observed that their participants tended to reject paradoxical versions of this inference and to accept modulated versions of it.
It is obvious that their results clearly support the mental models theory and, in principle, it can be thought that these prove that the mental logic theory is not correct. This last theory, as said, can explain why people reject the paradoxical versions, but not why they accept the modulated versions. Nonetheless, in my view, it is not true that the mental logic theory cannot explain why the participants in Orenes and Johnson-Laird’s (2012) experiment tended to admit the modulated versions. To show it, it is only necessary to assume that the logical forms of propositions are not exactly those that literally appear in natural language. In this way, my idea is basically that, in the modulated versions, the conclusion is not truly a disjunction.

4. **The Logical Form of the Modulated Versions**

The conclusions of the modulated inferences are only apparent disjunctions and it can be proved precisely by resorting to the analysis of possibilities that, based on the mental models theory, Orenes and Johnson-Laird (2012) raise. As indicated, the conclusions of the modulated versions only refer to models of the types (A) and (C), that is, to models with these structures:

\[(A) \ p \& \ q \]
\[(C) \ \neg p \& q \]

This means that the conclusions can only be true in one of these two cases: when both \(v(p) = 1\) and \(v(q) = 1\) or when \(v(p) = 0\) and \(v(q) = 1\). Obviously, ‘\(v\)’ refers to the truth value of the formula between brackets, ‘1’ stands for truth and ‘0’ stands for falsehood.

However, based on this information, it cannot be said that the conclusion is a disjunction, since as it is well-known, \(v(p \lor q) = 1\) if one of these situations happens:

1) \(v(p) = 1\) and \(v(q) = 1\)
2) \(v(p) = 1\) and \(v(q) = 0\)
3) \(v(p) = 0\) and \(v(q) = 1\)

Otherwise, \(v(p \lor q) = 0\).

Therefore, it seems suitable to think of a formula that is only true in cases (A) and (C), that is, a formula with a true table in which the result is 1 only in the cases \(p \land q\) and \(\neg p \land q\). It is clear that the formula can be, for example, \((p \lor \neg p) \rightarrow q\). In fact, this last formula seems to describe the exact meaning of Orenes and Johnson-Laird’s (2012) example, which appears to indicate that, both if Lucia wears the bracelet and if she does not wear the bracelet, she wears jewelry.
Thus, if we assume that the real logical form of the conclusions of the modulated inferences is not \( p \lor q \), but \( (p \lor \neg p) \rightarrow q \), the problem seems to be solved. It is true that, as the mental logic theory claims, people usually do not apply the disjunction introduction rule. What happens to the modulated inferences is that they are accepted because they do not require to use that rule, but other different rule, the conditional introduction rule \( (q \vdash p \rightarrow q) \).

Nevertheless, a new problem appears here. The conditional introduction rule is difficult to accept too. Orenes and Johnson-Laird (2012) also consider the inferences based on that last rule to be paradoxical, and they propose modulated versions for them as well. Besides, the conditional introduction rule is thought to be problematic even from the mental logic theory. For example, Braine and O’Brien (1998b) assign it a limited role in the basic model linked to primary skills of the mental logic theory, and Braine and O’Brien (1998c) acknowledge that they thought of the possibility of introducing some restrictions to this rule. Nonetheless, this new problem can be easily removed. The real true table of the conclusion of the modulated inferences is indeed equivalent to that of \( (p \lor \neg p) \rightarrow q \), but it is equivalent to that of only \( q \) too.

Thus, it is also possible to assume that the real logical form of the modulated conclusions is just \( q \). It would not be hard to admit because the underlying idea to Orenes and Johnson-Laird’s (2012) example also seems to be that, whatever happens, Lucia wears jewelry. But the most important fact here is that, if \( q \) is the true logical form of the conclusions of the modulated inferences, it can be stated that the real structure of the modulated inferences is not \( q, \text{ergo } p \lor q \), but \( q, \text{ergo } q \), or, if preferred, \( q \vdash q \).

In this way, given that it does not appear to be a problem, from the mental logic theory perspective, that a formula can be drawn from itself, it can be said that the mental logic theory can also explain the results achieved by Orenes and Johnson-Laird (2012) regarding the disjunction introduction rule. According to this theory, people do not usually apply this rule, which explains why the participants rejected the paradoxical versions (their structure was \( q \vdash p \lor q \)). On the other hand, the modulated versions tended to be accepted because their real structure did not refer to the application of the disjunction introduction rule (their true structure was \( q \vdash q \)). Undoubtedly, if it is assumed that the real logical forms of propositions are not necessarily linked to their structures in natural language, these arguments can be assumed too.

5. Conclusions
As said by López Astorga (2013), the problem of the current cognitive science is that there are several theories that can explain the same phenomena. Clearly, it is necessary a crucial information or obvious evidence that allows us to decide which of such theories is the correct one. Unfortunately, Orenes and Johnson-Laird’s (2012) research do not appear to provide that information or that evidence.

Certainly, the mental logic theory seems to need the additional assumption related to the existence of a previous phase of interpretation in which the logical forms are recovered. Equally, it is true that that recovery process depends, in a way, on the mental models theory, since it is based on the semantic analysis of the possibilities to which propositions can refer. So, it is clear that syntactic approaches cannot ignore semantics, and that this point reveals an important strength of the mental models theory, which does not seem to need to consider syntax in its accounts.

Maybe the correct alternative is not to choose between one of these theoretical frameworks. This is because it is possible to think that both theories are complementary and that each of them addresses a different aspect of cognitive activity. A thesis could be that the mental models theory describes the semantic processes in which the information is interpreted and the real logical forms are detected, and that the mental logic theory describes the subsequent processes in which certain logical rules (not necessarily all the rules of the classical logic) are applied to the recovered logical forms.

I am aware that the proponents of the mental models theory could argue against this idea claiming that their semantic analyses are enough to explain and predict the results obtained by individuals in most of reasoning tasks, and that, for this reason, it is not necessary to recover logical forms. Nevertheless, the precedent arguments show that syntax and semantics are not incompatible and that they can be strongly linked. In this way, although we can choose a semantic or a syntax approach, we should not forget that, apparently, cognitive phenomena could be explained from both perspectives.

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