Improving the Local Geoid with GPS

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Abstract. We may get coordinates over WGS84, for a set of points in a geodetic net, by using GPS receivers. But these are geometric coordinates, so they do not have the physical meaning we usually need in the most of our geodetic and geodynamic works.

From the global geoid we may get geodetic coordinates of those stations, by using interpolation. Then we can get the geoid undulation, and so the orthometric height. Now we have got a physical quantity to use in hydrographics, geographic information systems, airborne and shipborne navigators, geodynamics,... But the orthometric height accuracy will depend on the accuracy of the global Geoid Model.

The aim of this work is to present a working method in order to get the local geoid undulation, using GPS, absolute and relative gravimetric measurements and high precision levelling in a small geographic area. Such method may be useful in Hydrographics in order to get sounds in respect to the geoid.

1 Introduction

Ellipsoid models adopted as an aproximation to the Earth figure do not have the physical meaning that we need for geodetic applications like Geophysics, Geodynamics, Hydrographics, Precision Navigation, Geographic Information Systems... because height plays a definite role. For these applications should be more convenient to use the Geoid like a reference surface to define height systems due to its physical nature, because it is an equipotential surface, and because it is accessible through the mareographic registrations.

When you get the height over the Geoid for a point on the topographic surface of the Earth, you have located that point on a new equipotential surface. But this surface is not parallel to the Geoid surface and, of course it is not parallel to intermediate surfaces that we get during the geometric levelling works. This is the reason why the height that we get depends on the route we choose to get the geometric levelling. So we must not use geometric levelling to define geodetic heights.

But the difference of potential between two equipotential surfaces do not depends on the route we use to go from one of these points to the other. From this difference of potential, that we get making measurements of gravity and then geometric levelling to join intermediate points \( dW = -g \, dn \), we determine the Geopotential number:

\[
C_p = W_p - W_0 = - \int_0^P dW = \int_0^\infty g \, dn
\]

where \( W_p \) is the potential of gravity at P and \( W_0 \) is the Geoid potential of gravity.

The geopotential number is not given by length units but geopotential units called gpu's, so \( 1 \text{gpu} = 1 \text{KGal} \), \( 1 \text{KGal} = 100000 \text{cm}^2 \text{s}^{-2} \), then it is difficult to define a height system with such units. So we introduce the orthometric height \( H_p \) for a point P. It is the length along the plumb line from P to the point where the plumb line cross the Geoid. Let us link orthometric height with the geopotential number and the average gravity along the plumb line

\[
H_p = \frac{C_p}{g}
\]

with

\[
\bar{g} = \frac{1}{H} \int_0^H g \, dH
\]

We have an implicit equation, so to solve we need to adopt an approximate value of \( H \), besides the module of gravity vector in a set of points of the surface of the Earth.

The undulation of the Geoid in one point on the surface of the Earth is defined as the geometric length along the plumb line from the point, between the ellipsoid and the geoid surfaces.

In a local area we may approximate the plumb line with the vertical line above the ellipsoid. Then we get \( h = H + N \), with \( h \) as height above the ellipsoid, \( H \) as orthometric height and \( N \) as undulation of the Geoid.

Now it is obvious that in Geodetic applications where we need physical meaning we may get the local geoid
from the undulation using gravimetric measurements, geodetic levelling, and getting the ellipsoidal height from GPS positioning, because in a point \( P \) we get:

\[
N_P = h_P - H_P
\]

2 Method of computation

Let us consider a set of points, \( P_1, P_2, ..., P_n \), distributed over a local area, in such a way that for each point \( P_i \) we have got the absolute value of gravity, \( g_i \), geodetic levelling, \( n_i \), and ellipsoidal coordinates, \( \varphi_i, \lambda_i, h_i \).

We can get the orthometric height, \( H_i \) in each point subtracting that absolute values of gravity and geodetic levelling, from the ellipsoidal height. So we have got the experimental undulation of the geoid, \( N^\xi_i \) in that point as a first approximation.

On the other hand, we get the value \( N^\xi_i \) for the undulation of the Geoid from the interpolation in the model of geoid in use, at the point of coordinates \( \varphi, \lambda \).

Then we only need to compare both values \( N^\xi_i \) and \( N^\xi_i \) to derivate what is the correction that we have to introduce in the global model in order to get the local geoid.

2.1 Developing the Method

In order to develop the computing method we can follow [1] for instance. We are going to be able to calculate the value of the orthometric height for a point \( P_i \) in the local area, that we get from

\[ H_P = \frac{C_P}{\gamma_P}, \]

with \( C_P \) as the geopotential number and \( \gamma_P \) as the average gravity along the plumb line from the geoid to the point \( P \). Then the orthometric height difference from the point \( P_i \) and any other point \( P_j \) in the local area is:

\[ \Delta H_{P_i, j} = H_{P_i} - H_{P_j} = \frac{C_{P_i}}{\gamma_{P_i}} - \frac{C_{P_j}}{\gamma_{P_j}}. \]

Let us introduce the dynamic height \( H^{\text{din}} \) in order to compute the orthometric height difference between two points. We are going to define this quantity as the geopotential number for the point over the gravity constant. It is usual to take this gravity constant from the normal gravity value for the sea level at \( 45^\circ \) of latitude. You can get it as \( \gamma_{45^\circ} = 9.822652123 \text{ m s}^{-2} \) calculating from [6].

So:

\[ H^{\text{din}}_{P_i} = \frac{C_{P_i}}{\gamma_{45^\circ}} \]

Then we have:

\[ \Delta H_{P_i, j} = H_{P_i} - H_{P_j} = H^{\text{din}}_{P_i} + H^{\text{din}}_{P_j} + (H^{\text{din}}_{P_i} - H^{\text{din}}_{P_j}) \]

\[ = \Delta H^{\text{din}}_{P_i, j} + (H_{P_i} - H^{\text{din}}_{P_i}) - (H_{P_j} - H^{\text{din}}_{P_j}) \]

The dynamic height difference between \( P_i \) and \( P_j \) is:

\[ \Delta H^{\text{din}}_{P_i, j} = H^{\text{din}}_{P_i} - H^{\text{din}}_{P_j} = \frac{C_{P_i} - C_{P_j}}{\gamma_{45^\circ}} = \]

\[ = \frac{1}{\gamma_{45^\circ}} \int_{P_i}^{P_j} g \; dn = \int_{P_i}^{P_j} g \; dn + \int_{P_i}^{P_j} \frac{g - \gamma_{45^\circ}}{\gamma_{45^\circ}} \; dn = \int_{P_i}^{P_j} g \; dn + \int_{P_i}^{P_j} \frac{g - \gamma_{45^\circ}}{\gamma_{45^\circ}} \; dn \]

The integrate

\[ \int_{P_i}^{P_j} g - \gamma_{45^\circ} \; dn, \]

is called dynamic correction DC\( P_i, j \), between the points \( P_i \) and \( P_j \).

The difference between the dynamic and the orthometric height in one point \( P_i \) is the dynamic correction from the point \( P_i \) to that one \( P_0 \) that we get from the cross of the plumb line in \( P_i \) and the geoid

\[ H_{P_i} - H^{\text{din}}_{P_i} = \text{DC}_{P_i, 0} = - \int_{P_0}^{P_i} g - \gamma_{45^\circ} \; dH \]

And so:

\[ \Delta H_{P_i, j} = \int_{P_i}^{P_j} g - \gamma_{45^\circ} \; dn - \int_{P_i}^{P_j} \frac{g - \gamma_{45^\circ}}{\gamma_{45^\circ}} \; dn - \int_{P_i}^{P_j} \frac{g - \gamma_{45^\circ}}{\gamma_{45^\circ}} \; dH \]

In this expression \( dn \) indicates geodetic levelling and \( dH \) indicates integrates along the plumb line.

Levelling is a discrete process, so in the latter expressions, we may substitute the integrates for sums. And if we consider the average gravity that we have already introduced, we get:

---

Fig. 1. Geoid vs Ellipsoid
\[ \Delta H_{P_{1},i} = \Delta n_{P_{1},i} + \sum_{i=1}^{P_{1}} \frac{g_{i} - \gamma_{0}}{\gamma_{0}^{5^0}} \delta n_{i} - \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{1}} + \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{i}} \]

where

- \( \Delta n_{P_{1},i} \) is the geometric level difference between \( P_{1} \) and \( P_{j} \);
- \( \bar{g}_{P_{1}} \) and \( \bar{g}_{P_{i}} \) are the average gravity values at \( P_{1} \) and \( P_{i} \);
- \( g_{i} \) is the absolute value of gravity at the \( i \)-th intermediate point in the route from \( P_{1} \) to \( P_{j} \);
- \( \delta n_{i} \) is the geometric level difference from the intermediate point \( i \)-th to the intermediate point \( (i-1) \)-th in the route.

- \( H_{P_{1}} \) and \( H_{P_{i}} \) are the orthometric heights at the points \( P_{1} \) and \( P_{i} \).

We cannot make gravity measurements along the plumb line to compute the average gravity. Then we use the model called Poincaré and Prey reduction. Let us consider the point \( Z \) on the plumb line for \( P_{i} \), with an orthometric height \( H_{Z} \). The value of gravity for \( Z \) is:

\[ g_{Z} = g_{P_{i}} - \frac{\int_{Z}^{P_{i}} \frac{\partial g}{\partial H} dH}{2} \]

We are going to use the Bruns formula to compute the gradient of gravity below the earth's surface

\[ \frac{\partial g}{\partial H} = -2gJ + 4\pi k \rho - 2\omega^{2} \]

\( J \) is the average curvature for the geopotential surface; \( k \) is the gravitational constant; \( \rho \) is the density and \( \omega \) is the earth rotation angular velocity.

On the other hand the gradient of gravity for the normal gravity is:

\[ \frac{\partial g}{\partial h} = -2gJ_{0} - 2\omega^{2} \]

\( \gamma \) is the normal or ellipsoidal gravity, \( J_{0} \) is the average curvature for the spheropotential surface. Let us approximate \( gJ \) with \( gJ_{0} \) and we get:

\[ \frac{\partial g}{\partial H} = \frac{\partial g}{\partial h} + 4\pi k \rho \]

The gradient of gravity for the normal gravity could be expressed as a function of the latitude:

\[ \frac{\partial g}{\partial h} = -0.30877(1 - 0.00142 \sin^{2}\varphi) 10^{-5} \text{ m s}^{-2} \]

Let us now give the values \( \rho = 2.17 \text{ g cm}^{-2} \), \( k = 6.6710^{-9} \text{ c.g.s. units} \), and neglecting the variations of the gradient of gravity for the normal gravity with the latitude, i.e. the latitude of \( P_{1} \) and \( P_{10} \) is the same in a first approximation, results:

\[ \frac{\partial g}{\partial h} = -0.0848 \text{ Gal Km}^{-1} \]

and so:

\[ g_{Z} = g_{P_{i}} + 0.0848 (H_{P_{i}} - H_{Z}) \]

If we integrate this expression along the plumb line from \( P_{1} \) to \( P_{10} \) with respect to the orthometric height \( dH_{Z} \), we get:

\[ \bar{g}_{P_{i}} = \frac{1}{H_{P_{i}}} \int_{P_{1}}^{P_{10}} g_{Z} dH_{Z} = g_{P_{i}} + 0.0424 H_{P_{i}} \]

\( g \) is given in Gals and \( H_{P_{i}} \) in kilometers, but as levellings are usually given in meters, it is more convenient to use:

\[ \bar{g}_{P_{i}} = g_{P_{i}} + 0.0424 \cdot 10^{-3} H_{P_{i}} \]

because \( H_{P_{i}} \) is given now in meters.

Now we are going to introduce for the orthometric height difference between \( P_{1} \) and \( P_{j} \) the expression:

\[ \Delta H_{P_{1},j} = \Delta n_{P_{1},j} + E_{P_{1},j} \]

\( E_{P_{1},j} \), is called orthometric correction and is given by:

\[ E_{P_{1},j} = \sum_{i=1}^{P_{1}} g_{i} - \gamma_{0}^{5^0} \delta n_{i} + \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{1}} - \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{i}} \]

Let us substitute \( \bar{g}_{P_{1}} \), and we get:

\[ \Delta H_{P_{1},j} = H_{P_{1}} - H_{P_{j}} = \Delta n_{P_{1},j} + \sum_{i=1}^{P_{1}} g_{i} - \gamma_{0}^{5^0} \delta n_{i} + \frac{g_{P_{j}} + 0.0424 \cdot 10^{-3} H_{P_{j}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{j}} - \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{i}} \]

then:

\[ \Delta n_{P_{1},j} + \sum_{i=1}^{P_{1}} \frac{g_{i} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} \delta n_{i} + \frac{g_{P_{j}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} H_{P_{j}} + 0.0424 \cdot 10^{-3} H_{P_{j}} - \gamma_{0}^{5^0} H_{P_{j}} - (1 + \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}})H_{P_{i}} = 0 \]

and so:

\[ \frac{0.0424 \cdot 10^{-3}}{\gamma_{0}^{5^0}} H_{P_{j}} + \frac{g_{P_{j}}}{\gamma_{0}^{5^0}} H_{P_{j}} + T_{P_{i}} = 0 \]

with

\[ T_{P_{i}} = \Delta n_{P_{1},j} + \sum_{i=1}^{P_{1}} \frac{g_{i} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} \delta n_{i} - \left(1 + \frac{\bar{g}_{P_{1}} - \gamma_{0}^{5^0}}{\gamma_{0}^{5^0}} \right) H_{P_{i}} \]

So if we can get at the points \( P_{1} \) and \( P_{j} \) the values of geometric levelling, and absolute gravity, and the orthometric height for the point \( P_{i} \) we can solve the quadratic equation and get the orthometric height for \( P_{j} \).

We are going to get the geoid undulation at \( P_{j} \) subtracting from the ellipsoidal height \( h_{P_{j}} \), that we have from GPS positioning, the orthometric height that we have computed:

\[ N_{P_{j}} = h_{P_{j}} - H_{P_{j}} \]
3 Application of the Method

For the application of the method we need the values of the orthometric and ellipsoidal height at each one of the points in the local area.

From GPS positioning we get ellipsoidal height with respect to WGS-84 reference ellipsoid. But we also need a geodetic vertex with accurate coordinates to adjust the position of the selected points.

To compute the orthometric height in one point we need to know the absolute value for the gravity and the geometric level difference between this point and one reference point where we know the absolute gravity value. We need to determine absolute gravity value at the intermediate points in the levelling route too.

The determination of the absolute value of gravity is a complicated work. The tools that you need have to be isolated and so they have to get a good stability. Then you find that you cannot place your absolute gravimeter everywhere, and so you have to choose what stations have to be observed with that tool. To get the gravity everywhere we have to use relative gravimeter. So, if you know the absolute gravity in some place, you are going to refer your measurements to that point. You have to begin in such point that you know the orthometric height, ellipsoidal coordinates, and absolute gravity.

The local area that we choose to implement the method is the Real Instituto y Observatorio de la Armada (Royal Spanish Naval Observatory). In the following we are going to call it ROA. There we have a main point with its geodetic coordinates well defined, an absolute gravity valued point, a relative gravity calibration station, and a precise geodetic GPS, included in EUREF89 net.

3.1 Geodetic GPS net

A geodetic GPS net is a set of stations performing simultaneous tracking of GPS satellites. From the data process we get a high precision relative positioning among the stations. To refer this net to a major geodetic net you need that at least one of its points belongs to it, i.e., you need to know the precise geodetic coordinates of such point, and so you are able to adjust the observed net, introducing the coordinates of the known station as fixed coordinates. Now you get precise coordinates for each point you have observed.

During the first stage of the campaign EUREF, European REFerence, in 1989 it was chosen a point at ROA, and so we have a precise GPS point there. This point is what we have used as the main station for our work.

We have chosen eight points, including ROA for our geodetic net. We have called them as Euref, that is precisely ROA, together with Hora, SLR station, Muro, Pilar NE, Pilar NW, Astrolabio and IGN-135.

We perform simultaneous observations at the main station and at each one of the stations, and we get a set of baselines, with geodetic receivers TRIMBLE 4000 SST. Each session had a minimum of 2 observation hours, and of course we have always at least 4 satellites, and an elevation mask of 15°.

We use a short length model to reduce the baselines, because no one of them is bigger than one kilometer. We believe that we are going to get enough accuracy for such short baselines using the TRIMBLE NAVIGATION data process software GSUSRVEY, instead of a scientific one. The solving model used by this software performs the usual double-differences algorithm from two receivers to two satellites. The ionospheric effect is neglected due to the short length of the baselines, and even the tropospheric effects can be neglected. We get the coordinates for the new stations in a toponometric local reference system, i.e. we get differences in distance, Δd, azimuth, Δθ, and height, Δh, from the main station.

In order to get now absolute geodetic coordinates you have to apply a least square with constraints adjustment. From the internal accuracy of the observations you are able to get the weights you have to apply in the adjust.

The Table 1 shows the absolute coordinates for the geodetic net observed, and the values of the standard deviations for the observed baselines from the EUREF station to each of the stations.

The latitude is written in arc seconds over 36°27', the longitude is written in arc seconds over 6°12'. Heights are given in meters

3.2 Gravimetric measurements

The SELF project, is an European research project developed for a set of institutions, and supported by the European Union. ROA is one of the institution that is scheduled to participate during the second phase of the project, if it is approved. During the first phase, when ROA did not participate, the Institut für Angewandte Geodäsie (IFAG) came to ROA and with our collaboration chose here one suitable point to compute the absolute gravity. It is located in a tunnel where the ROA long period sismometric instruments are placed. There were performed absolute gravity measurements using an absolute gravimeter FGS-Axis. The value that they got was 9.796267065 m s⁻². There was located another point to calibrate relative gravity measurements, using a relative gravimeter Lacoste-Romberg LCR D-21, from the...
absolute point. The difference with the absolute gravity value they computed was $1.9386 \pm 1.9 \mu \text{Gal}$.

From these data we use in our field work relative gravimetric observations in each one of the above mentioned stations, with a relative gravimeter W. Sodin-200.

The accuracy for the lectures is $10 \mu \text{Gal}$. We added in the gravimetric observations two intermediate stations, that unfortunately are not accessible to instale GPS antennas. These stations were located at the ROA main building access, and at the ROA meteorological tower access. The table 2 shows the absolute gravity values and the standard deviations for each one of the geodetic net points.

<table>
<thead>
<tr>
<th>Station</th>
<th>Gravity (mGal)</th>
<th>$\sigma$ (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGN (1935)</td>
<td>978820.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Astrolabio</td>
<td>978825.66</td>
<td>0.02</td>
</tr>
<tr>
<td>Euref</td>
<td>978822.27</td>
<td>0.03</td>
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<tr>
<td>Calibration</td>
<td>978824.78</td>
<td>0.02</td>
</tr>
<tr>
<td>Hora</td>
<td>978823.96</td>
<td>0.03</td>
</tr>
<tr>
<td>Muro</td>
<td>978820.45</td>
<td>0.02</td>
</tr>
<tr>
<td>SLR</td>
<td>978816.66</td>
<td>0.02</td>
</tr>
<tr>
<td>NW</td>
<td>978820.08</td>
<td>0.05</td>
</tr>
<tr>
<td>NE</td>
<td>978820.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Main Build.</td>
<td>978824.42</td>
<td>0.05</td>
</tr>
<tr>
<td>Met. Tower</td>
<td>978824.89</td>
<td>0.06</td>
</tr>
</tbody>
</table>

3.3 Levelling

We used a level Wild NAK2 and one double millimeter aluminium stave to perform the levelling works, that included the next points Astrolabio, Euref, Hora, GPS, Relative Gravity Calibration Point, Main Building Access, Terraza, Muro, Pilar NW, Pilar NE, SLR station and IGN-1935. The point IGN-1935 is included in the First Order Spanish Geodetic Net. So we have its orthometric height, 43.154 m and we take it as the fundamental point for the heights. And as we cannot make a route to go from the point EUREF to the SLR station, we made a geodetic levelling with a Wild T2000 theodolite from two control points previously established linked with the rest of the levelling stations.

The level differences with the point IGN-1935 are shown in the table 3.

<table>
<thead>
<tr>
<th>Station</th>
<th>$\Delta h$ (m)</th>
</tr>
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<tbody>
<tr>
<td>Astrolabio</td>
<td>+17.792</td>
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<td>Euref</td>
<td>+7.890</td>
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<td>Calibration</td>
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<tr>
<td>Main Build.</td>
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<tr>
<td>Met. Tower</td>
<td>+2.020</td>
</tr>
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4 Results and Conclusions

GPS, gravimetric and levelling geodetic nets of ROA have common points with ellipsoidal geodetic coordinates, absolute gravity and geodetic levelling. So we are able to apply the above described method, and to compute the geoid undulation at such common points. We have used interpolation methods to get the values of the geoid OSU91A for the local area, and so to compare both set of results. We concluded from the results that if we need precise heights to use in that applications that we have already mentioned, we have to use the method described in this article due to the significant differences with the theoretic model. A very important application of this work will be devoted in the future.
to hydrographic works. The National Hydrographic Services refer the depths that they get for their charts to their own Hydrographic Zero. It means that the depth could not coincide in different charts from different hydrographic services. To homogenize all those different national reference systems an international agreement, that is nowadays under discussion, have to be suscribed. The National Service have to change its level reference to the one adopted. It could be for instance, at this is one of the points under discussion, the level of the Geoid. So the accuracy of its determination has to be improved. It is dramatically true in those areas where the depth is not so big to ensure a safety navigation, for instances areas near the coast, or port areas. There you may find, using the procedure we have used in our work, an easy way to determine the geoid for a set of points, that allows you to get the local geoid to build your chart.

<table>
<thead>
<tr>
<th>Station</th>
<th>OSU91A</th>
<th>ROA95</th>
<th>dif.</th>
</tr>
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<tbody>
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<td>44.424</td>
<td>1.363</td>
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<td>Astrolabio</td>
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<td>45.794</td>
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<td>IGN (1935)</td>
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<td>44.370</td>
<td>1.421</td>
</tr>
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</table>

References


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Editors' introduction

In this volume you will find the Proceedings of the Session G7 "Techniques for local geoid determination", held on Wednesday 8 May, 1996 at the European Geophysical Society XXI General Assembly in The Hague, 6 - 10 May, 1996. We had the pleasure to serve as the conveners of this session, which was well attended and where many new results and several interesting and stimulating theoretical procedures were presented. The session was divided into three oral sub-sessions and one poster sub-session, which were chaired by the two conveners and Dr. Nikolaos Pavlis.

This report appears well before the next EGS meeting on April 1997 in Vienna, where also the editors of the present volume will convene the session G12 "Developments in spectral and stochastic techniques for gravity field modeling", continuing thus a good tradition.

We sincerely hope that the reader of the present volume will find it a good reference in gravity field modeling researches and applications. We arranged the articles according to the order of their presentation in the meeting. For a number of papers the abstracts have only been included, either because the full papers will be published in different scientific journals or because the authors did not submit their contributions in time.

The first editor is grateful to the Finnish Geodetic Institute for support and providing its facilities in publishing this Proceedings volume.

We would like to thank the Organizing Committee of the General Assembly for providing this framework, and also the contributors to Session G7 for their effort and help, and we hope we meet again in Vienna!

Thessaloniki/Masala, 30 August 1996
Ilias N. Tziavos and Martin Vermeer, editors