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FROM TRADITIONAL TO COMPUTATIONAL ARCHEOLOGY.

AN INTERDISCIPLINARY METHOD AND NEW APPROACH TO VOLUME AND WEIGHT QUANTIFICATION

ABSTRACT:

The present study aims to show the effectiveness of a methodological procedure to estimate the volumetric capacity of archaeological ceramic vessels and the net and gross weights of their probable contents. This method can be easily applied, independently of cultural or chronological contexts, and could serve to verify the possible existence of processes of typological and metrological standardization of any domestic or commercial container, not to mention other possible historical and economic conclusions. This study gives a detailed description of a simple methodological protocol which uses profile drawings to calculate the approximate volume of any vessel, thus enabling its conformity to ancient weight systems to be assessed. This article will illustrate the strength of the method using a sample of Ramon T-11213 amphorae made in the Bay of Cadiz during the 5th century BC which, given its quantitative and qualitative strengths, proves to be an exemplary case study and a valid pilot.

KEYWORDS:

Computational Archaeology; Ceramic vessels; Volume; Weights; Mathematical Methods; Numismatics.

1. INTRODUCTION

The purpose of this paper is to present a methodological protocol, based on simple and easily reproducible archaeological and mathematical procedures, to estimate, with little margin of error, the capacities (volumes) and weights (gross and net) of any vessel or transport container made in Antiquity, starting only with a profile drawing. This protocol is based on the contributions of a team of members trained in archaeology, mathematics and engineering, in an interdisciplinary collaboration that results in the proposal of a proven, verified and scientifically well-founded procedure.

Through an analysis that goes beyond traditional, typological observations, this article outlines a methodological protocol that uses the data obtained by computational software (graphical representation, mathematical and statistical calculation), to calculate (indirectly) the values of possible standards of volume and weight, the conclusions of which would be extremely useful in the study of trade and economy in ancient societies. Starting from this empirical approach, it searches for a model applicable to any historical period in any geographic-cultural context. In summary, it is intended to offer a methodology that allows for the study of the typological, morphometric and volumetric standardization of any vessel in relation to the system of weights and measures commonly used in each particular area of study.

It should be noted that the approximations of ancient weights and measures have thus far mainly been approached from the theoretical analysis of the references of classical literary sources (Powell 1992; Pellicer 1997; Teodor 1998, 2000, 2000a; Zamora 2003) and not from experimentation, direct or digital. Furthermore, the study of the metrological patterns of Antiquity remains an underexplored area with great potential, due to the fact that the multitude of metrological references and their fractioning are not yet clear. In fact, as proven by recent studies (Moreno and Arévalo 2017), the research of ancient systems of weights and measures allows us to pursue, with interesting results, lines of parallel or related research which have hitherto been less accessible, but which will be discussed in this study.

This article describes in detail a valid methodological process, translatable to any cultural and chronological field, the effectiveness of which will be demonstrated with data obtained from recent quantification studies of containers manufactured in Gadir (Cadiz, Southern Spain) in the fifth century BC. This concrete example substantiates that the volumetric and weight investigation of ancient containers in the archaeology of Cadiz is still developing. Nevertheless, this kind of computational analysis is generating advances in fields such as the use and diffusion of patterns of volume and weight, as well as changing relationships with the introduction and fixation of the monetary system (Moreno and Arévalo 2017). This procedure also allows for the calculation of volume production and better estimations of the overall magnitude of these industrial-commercial manifestations in the economic history of any region under analysis (Sáez and Moreno 2017).

The case study centres on the analysis of transport containers for trade in the Punic Bay of Cadiz, with the focus on a sample of twenty-five complete or nearly complete local T-11213 amphorae dating to around the 5th century BC. The choice of this group of archaeological items and this particular geographical and historical scenario can be attributed to two factors. Firstly, the enormous importance of the ancient city of Gadir as the main political, cultural and economic focus of the South of the Iberian Peninsula and secondly, the fact that Gadir was the leading centre in the region in the use of monetary economic systems, as the city had the first mint in the area in the beginning of the third century BC. Further justification for the choice is that currently, these archaeological materials are optimally systematised (Ramon 1995). Also, there is a large set of complete containers accurately and chronologically classified that, methodologically, provides a synchronous representative sample that can yield statistically significant results (Sáez and Moreno 2017).

2. FROM THE ARCHAEOLOGICAL RECORD TO MATHEMATICAL CALCULATIONS. PREVIOUS INTERDISCIPLINARY EXPERIENCES AND APPROACHES.

Despite the intense development that digital technology and computer software have experienced in recent years, this progress has not yet had a commensurate impact on overall methodological applications in archaeological research. Even with its increasing diffusion and methodological implementation, the use of these new technologies in the documentary processing of archaeological objects still presents a major challenge. Moreover, paradoxically, we must admit that there is still a shortage of interdisciplinary teams made up of archaeologists, mathematicians and engineers. Due to this, the work carried out in archeology in this computational line still far from rivals that done in other areas, such as the purely typological ones.

As an exception, the renowned work of the LAQU (Laboratori d'Arqueologia Quantitativa) project of the University of Barcelona, focuses on the theoretical and methodological development of digital techniques for its computer application in Archaeology (Barceló 2007, 2009; Barceló and Vicente 2011), encouraging the development of new approaches to the joining of Mathematics and Archaeology (Barceló and Bogdanovic 2014). In this innovative line of study, Professor Uzy Smilansky's Laboratory of Computer Archaeology (Karasik et alii 2004a; Karasik et alii 2004b; Sabatning et alii 2005; Sergi et alii 2012; Gilboa et alii 2013; Grosman et alii 2014) from the Weizmann Institute of Science in Israel leads the way in the development of mathematical and computerized methods for archaeological research. The team has undertaken work on the volumetric principles, typology and classification of ancient ceramics, either from their profiles or by methods of precise orientation of the ceramic fragments for subsequent drawing and reconstruction.

Nevertheless, it should be noted that the approach to the calculation of the capacities of ancient ceramic vessels remains a relatively unexplored area, owing to difficulties resulting from the archaeological record itself and further aggravated by the small number of complete items available. This has resulted in a general shortage of studies devoted to volume and weight, and also those which seek to analyse the processes of diachronic evolution of old metrological systems from data obtained through the computational analysis of archaeological pots.

Despite the lack of investigation, the importance of these quantitative data to the study of Economic History has not gone unnoticed in international research. In fact, it is a line that has been developing for several decades, especially in Anglo-Saxon literature (Lang and Crosby 1964; Matheson and Wallace 1982; Wallace and Wallace 1982; Wallace 1984; Louise and Birnie 1994; Louise and Dunbar 1995; Senior and Birnie 1995; Anderson 1995, 1995a; Thomas and Wheeler 2002; Engels et alii 2009; Cohen et alii 2013). Furthermore, as previously highlighted, there have been interesting contributions from Israeli research in recent years (Van Alfen 1996; Karasik and Smilansky 2006; Karasik et alii 2006; Zapassky et alii 2006, 2009; Kletter 2009, 2014), and it should be added that the Russian archaeological school has also made remarkable progress in this area (Teodor 1998, 2000, 2000a; Monachov 2005; Vodolazhskaya 2008).

The main objective of these studies has been the search for a general method of volumetric calculation of ancient ceramic items, an issue which has for years been a priority, solidified with the emergence of groups formed of researchers from the fields of humanism, mathematics and engineering combined (Rodríguez and Hastorf 2013). Particularly remarkable are the initiatives aimed at developing a free and universal computer program for the automatic calculation of the capacities of these containers, such as the *Kotyle* computer application for measuring vessel capacity, accessible only to registered users (see kotyle.readthedocs.org) (Costa 2013).

Finally, it should be emphasized that, despite the formulation of interdisciplinary initiatives for the development of automated applications for quantifying the volume of containers of archaeological origin, due to factors such as the difficulty of the existing methods, these techniques are currently a long way from being included in a standardized manner in conventional Archeology research methods.

3. FROM LABORATORY TO AUTOCAD®: SELECTION, PROCESSING AND DIGITAL DOCUMENTATION OF ARCHAEOLOGICAL MATERIAL.

The methodological protocol proposed by this study differs from those described above, in that is has proven itself to be a fast and relatively easy system, requiring only a two-dimensional archaeological drawing of each item which is then digitized for further processing and analysis. This is advantageous as it allows the volume of the container to be quantified using only the existing bibliography, thus simplifying the laboratory work required to gain the primary information. However, it is imperative to keep in mind that the data collected from these secondary sources will lack accuracy in terms of origin and quality, a point which must therefore be fundamental in the estimation of the level of uncertainty in the measure of the results of the subsequent calculations.

The proposed method typically selects the pieces of the archaeological record most suitable for their metrological study, giving priority to those containers which are complete or almost complete. After collecting typological and morphological data and cataloging the chosen pieces, they must then be photographed and a traditional 2D drawing rendered of the profile.

In addition, and specific to this procedure, it is necessary to acquire accurate data on:

Dimensions:

Maximum and minimum body diameters.

Maximum diameter of the mouth.

Maximum conserved length.

Estimated non-conserved length.

Exterior appearance.

Density and thickness of the paste.

Technical or typological peculiarities.

Weight of the empty item.

Other observable data.

All of these are factors for controlling the method of calculation and digitization. The gathering of this information is essential, especially in the case of complete or nearly complete items, since this data is of particular importance when reconstructing the capacity and weight of these specimens. If such specific data as the weight of the empty vessel or the density of the paste are not recorded in the laboratory, the results obtained *a posteriori* on the typified (net and gross) weight of the containers will be highly uncertain and will be subject to an error the significance of which will be difficult to assess.

The process must be repeated for each of the selected types, thus processing the largest number of copies of each typological group, in order to have a sufficient amount of data with which to carry out a statistically significant evaluation of the capacities and average weights per standardized type. This documentation will be archived in individualized files created for this purpose, including a high resolution scan of the 2D paper drawings of the selected ceramics.

The next step will be to digitize the scanned drawings from the laboratory or from the published bibliography. For this purpose, the AutoCad vector drawing software has been proven to be a powerful set of 2D and 3D design tools, placing it as the most accredited digital design software both for drawing in engineering and architecture and as a tool for generating the type of precise two-dimensional drawing valid for use in archaeology.

AutoCad allows for the importation of the scanned drawing into its work area and, subsequently, the curve of the inner profile of each of the selected containers can be vectorized using the image as a base or template. For digitization, the axis of revolution of the item is drawn and guiding lines are placed at the beginning and end of the profile of each of the vessels, with the possibility of using the drawing method based on splines (soft curves) (Fig. 1).

At both the laboratory and AutoCad stages, special care must be taken with the reconstruction of any non-conserved curves when drawing the profile of each fragmentary vessel. A lack of precision in the profile will introduce inaccuracies into the reconstruction of the complete vessel according to its typological group, undermining or even invalidating the data thus obtained. In order to avoid this situation, when estimating the reconstruction of each vessel, a typified curve, previously obtained by the average profile of the largest possible number of complete vessels belonging to the same typological group, must be accurately followed. In addition, as is usual and logical in the collection of any scientific data, it is imperative to carefully distinguish between the reconstructed and the conserved data in order to make valid conclusions and discuss the results. (Fig. 1).

Once the vessel is digitized, x / y coordinates are taken from a series of highlighted control points at standard intervals along the inner profile of the vessel under study. To do this, at regular distances (from *c*. 5cm), lines are plotted perpendicular to the axis of revolution defined by the base and mouth of the object, marking and annotating the coordinates x / y of each control point (Fig. 2). These x / y coordinates will be obtained at *Cad* points, which must subsequently be scaled according to the measurements of each pot in cm or mm (Fig. 3, Table 1).

Regarding the calculation of the volume held by the containers, it is assumed that it would not completely reach the mouth, but would leave an empty space for sealing by various resins or opercula. Therefore, it is generally estimated that the maximum filling of each container would reach the end of the shoulder and the start of the mouth. The last control point, which measures the maximum estimated length for the calculation of the capacity of each container would, accordingly, be placed at this point. (Figs. 1 and 2). As this measure is not certain and is obviously relative, it should be as standardized and invariable as possible throughout the sample. Moreover, it should be taken into account when considering the mean of the maximum lengths of the sample and the possible error in the volume and weight calculation of each vessel.

The digital drawing and the gathering of the control points for the x / y coordinates of the internal profile is systematically repeated throughout the sample of selected items (Table 1), in order to create a sufficiently diverse data set for its later statistical consideration. Likewise, for the sake of greater transparency in the treatment of the figures, the fragmented receptacles will include both actual (conserved) and estimated (reconstructed) data.

The maximum diameter of each specimen should also be noted, so that individual measurements can be compared against averages within groups as part of a typological evaluation. The range of values for the maximum conserved and reconstructed lengths must be recorded, since the relationship between the length and diameter of a vessel obviously determines its volume. This relationship was well known in the pottery workshops, as evinced by ancient sources such as the Egyptian formulas of the mathematical papyrus of Rhind n. 41-47 preserved in the British Museum, and the Greek formula of Heron (Vodolazhskaya 2008). Although the application of formulas relating lengths, diameters and volumes is still controversial (Kletter 2009), they must be taken into account in order to try to advance the understanding of the processes followed by the artisans in the manufacturing of standard vessels in the pottery workshops, as they had to have undergone some kind of control and regulation, the nature of which, to this day, remains unclear.

4. FROM AUTOCAD® TO WOLFRAM MATHEMATICA®: INTEGRAL CALCULATION OF VOLUME OF REVOLUTION

The coordinates obtained by the method described in AutoCad, or in an alternative vector-drawing program, will be transferred to the computational software, Wolfram Mathematica, or to another calculation program such as the well-known SAGE. In order to determine the capacities of the selected ceramic vessels, a relatively simple mathematical method is employed using the conveniently scaled control points of the two-dimensional digitized profiles to obtain a vector spline (interpolation of a smooth curve from the defined points) of the interior of the object under study, from which its hypothetical volume can then be estimated in cm^3 (Fig. 3).

For calibration purposes, in order to both verify the validity of the method and to estimate the relative error of the procedure, a semicircle with a 50cm radius was drawn in AutoCad (Fig. 4) following the previously described systematic procedure. Only ten control points were considered and used for the Mathematica program (Fig. 5). The volume calculated by the software for the sphere of revolution presented a relative error of the order of +/- 1%, validating the method.

This shows that, from the first step of taking data from the internal profile of the vessel to its subsequent use in the calculation of the volume, there is hardly any error. This validates the procedure and assures that it can be transferred to any vessel produced by revolution with great accuracy. This calculation will be systematically repeated for all the specimens studied, in order to create a sufficiently contrasted data set for later statistical consideration (Fig. 6).

The method of digitization and subsequent calculation used in the pilot study, had already been utilised in previous studies (Moreno and Arevalo 2017; Sáez and Moreno 2017). The volumetric capacities were estimated for 25 complete or nearly complete amphorae specimens of the series T-11213 manufactured in Gadir in the fifth century BC (Table 2). Interesting data was obtained for each of the containers, the average volume being estimated at 52.93 litres.

5. FROM THE VOLUME TO THE WEIGHT: HOW TO CALCULATE MASS?

Once the capacities of each of the studied vessels have been obtained, the method is faced with the obvious problem that it is not possible to estimate the weights, understood here as synonymous with mass (calculated in kilograms) rather than as force (mass by acceleration of Gravity, expressible in Newtons), of full or empty pots based only on the calculated volumes. This presents a difficulty as the weight of each vessel will vary according to the density of the contents. Therefore, in order to estimate the gross weight of these receptacles, it will be necessary to know the density of both the content and the container, since it is well-known that mass (weight) is equivalent to volume * density (specific weight).

As for the content, therefore, the calculated estimation will vary according to the available data about what could have been carried by the vessels in analysis. It will depend on prior research and also on the data collected by Archeometry and the classical texts on the subject. In previous papers on the estimation of the weights of T-11213 amphorae manufactured in Gadir (Moreno and Arevalo 2017; Sáez and Moreno 2017), it was assumed that these containers were usually reserved for the transportation of salted fish in a 50/50 mixture of salt and fish broth, this being a rather conservative estimation based on the information provided by literary sources. Therefore, the individual net weights (Table 3) of each of the containers (Fig. 7) could be estimated by multiplying half the volume of each vessel by the density of the salted fish (estimated at 1.058 kg/l) and the other half by that of the salt (estimated at 1.1 kg/l).

Weighing complete or nearly complete empty ceramic vessels in the laboratory is of enormous importance in estimating the average tare weight and is fundamental to any gross weight quantification study. However, the archaeometric studies do not currently focus on the calculation of the density of the ceramics, which is essential to allow for the estimation of the average tare weight by calculating the difference of the volumes obtained by the application of the same described method of the spline to the external and internal profile of each vessel.

In the case study of Gadir amphorae T-11213, the weight of a single complete empty amphora (G), 14 kg, was known. This weight was also evidenced among the items published by Zimmerman-Munn (2003) from Corinth and, in the absence of any other data, this was estimated to be the average weight of the tare of the selected amphorae. However, in order to calculate the tare more precisely, and in view of the maximum diameters and lengths of the sample items, it was clear that the larger the size and capacity of the amphorae, the greater the weight would be. Therefore, an estimated ratio of the tare was calculated from the weight and volume ratio of each of the samples, thus obtaining a series of weights approximate to the real ones, though clearly not testable. Even so, the mean results of the gross weights of the sample calculated on the basis of the hypothetical average tare of 14 kg (a mean of the gross weight of 71.05 kg) or the calculated tare weight (an average of 69.82 Kg), were very similar, with both hypotheses being proposed as approximate but equally valid (Table 4).

6. FROM STATISTICS TO THE PATTERN: THE RECONSTRUCTION OF OLD METROLOGICAL SYSTEMS.

New hypotheses in the study of ancient metrological patterns can be discussed through the contrast of the available literary and archaeological sources with the data calculated by the method presented here. On these lines the diachronic evolution of the patterns, as well as its relation to the monetary and weight system in each cultural area under study, is remarkable. However, this is still a developing area of study, the results of which would allow for a better understanding of the stage of exchanges before the invention of the coin and the bases of the process of monetization of the old economies.

The application of the described protocol in particular cases where archaeological contexts, written testimonies and other historical evidence are known, provides for the advancement of new hypotheses regarding the reconstruction of the ancient metrological (weight, volumetric, longitudinal and monetary) systems, the measure of which varied culturally and chronologically. Consequently, the procedure followed in our case study serves as a valid example and as a pilot for transferring the protocol to other environments and historical periods.

According to recently obtained data (Moreno and Arevalo 2017), the averages of the sample volumes of amphorae analyzed in our case study seem to fit very well into the Hebrew volumetric pattern; a pattern which involves connections adjusted with a hypothetical previous system (Ugaritic) which was used in the Syrian-Palestinian coast at least from the Final Bronze, and would later have been exported to Gadir by the Phoenicians.

The volumetric pattern used in Ugarit is not yet well known. However, recent contributions (based on methods of quantification and mathematical calculation of the volume contained in the amphorae *lmlk* from Canaan) seem to present sufficient evidence to propose the possibility that the Ugaritic pattern was based on the weight of the volume of water measured according to the talent, thought to be 28.2kg (Pellicer 1997; Parise 2006). This is based on several archaeological testimonies, particularly regarding the weight of the ingots carried in wrecks such as Uluburum or Cape Geledonya (Petruso 1992).

The manufacture of the Canaanite amphorae could also have followed a standardization procedure based on the capacity of the *kd*, the measurement of which is estimated at between 12 and 14 litres and appears to theoretically approximate the intention to contain 14.1 litres of liquid content. The measure of the *kd*, in practice, would support the relationship between weight and volume ratified in the old processes of measurement (Pellicer 1997), since it would weigh half a talent. However, this Ugaritic pattern has not been sufficiently compared with epigraphic and archaeological sources (Zamora 2003), and therefore remains hypothetical, awaiting further justification. It would be interesting for there to be more studies of the volumes and weights of the amphorae found in the winery of Minet-el-Beida, the port of Ugarit, based on the application of the methodological protocol outlined here.

Unlike the Ugaritic pattern, the Hebrew metrological pattern was reconstructed by Hultsch in 1882 and later revised by other researchers, Docter (1988-1990) and Pellicer (1997), among others, based on the testimonies provided belatedly by Flavius Josephus (*Ant*. VIII. 2. 9). In spite of the greater reliability of this data, the potential evolution of the pattern, which is only known in diachronic terms, must be taken into account. In addition, the possible accumulation of errors in the proper reconstruction of these measures and their transfer to other systems must also be considered; Flavius Josephus already gives the equivalencies of the Hebrew magnitudes in terms of Latin weights and volumes. All these issues would eventually explain some of the differences between the theoretical values and those actually calculated.

Taking into account all of these difficulties, it can be said that the ancient Hebrew system of capacity measures was based on the dimension of the *koros* (393.81 litres), which was subdivided into 30 *sata* (1 *saton* = 13.127 litres), 180 *kabs* (1 *kab* = 2.188 litres) or 720 *logs* (1 log = 0.547 litres). The measurement of the *saton*, it can clearly be seen, is very close to that proposed hypothetically for the *kd*. A fact which is unsurprising given the connections and equivalences between both systems which were so close, both culturally and geographically.

As previously noted, the average volume of the Gadir amphorae T-11213 approaches 53 litres. When reducing this measure to *sata*, it is found that these amphorae could have been manufactured with the intention of containing an average capacity of almost 4 *sata*. This result clearly seems to corroborate that these vessels were manufactured in Gadir in the 5th century BC in a normalized form. This standard was related to a pattern of official measures with very close equivalents to the Hebrew system used in Judea in the first century AD.

However, the sample of amphorae in our case study had a standard deviation of 7.05 (Table 4). This encouraged more focus on statistical study, dividing the sample with respect to established ranges from the more reliable data, offered by the approximation to the capacity of full amphorae. This resulted in the finding that there were at least four different groups of capacity readily expressible in 4.5; 4; 3.5 and 3 *sata*. Therefore, these containers would have been manufactured in a standardized way following the quoted pattern and with different sizes to meet the different market needs.

Also, thanks to this methodology and to its statistical evaluation, observations can be made regarding the net and gross average weight of the studied amphorae. In this case, it was assumed that, according to the weight system used for the emission of its first coins, Gadir knew the metrological pattern of Ugarit, which was based on the talent of 28.2kg. This question had been previously addressed by García-Bellido (2003, 2013), based on the weight of the silver *hemishekels* issued in Gadir at the beginning of the third century BC. These *hemishekels* had an average weight of 4.5/4.7 g and were adjusted to the *shekel nsf* also referred to as Syrian, micro-Asian or Ugaritic (Elayi and Elayi 1997; Parise 2006) weighing 9.4g. Moreover, this *shekel* testifies to the use of a complex metrological pattern, equivalent to 3,000 parts of a talent of 28.2 kg, or to 50 parts of a minē of 470 g.

Thanks to the weight of the Gadir coins, it had been demonstrated that this system was well-known in the city. However, this data had not been contrasted with the hypothetical average weights of (or measured for) these amphorae (Table 4), since until very recently this information remained unknown. Thanks to this methodology it was possible to clearly observe that the average net weights of the T-11213 amphorae were very close to the weight of two talents (Table 5), which corroborates the usefulness of the described method and clearly expands its possibilities for historical research. Moreover, it was found that the average weight of an empty vessel was close to half a talent, a discovery which should lead to a rethink about the manufacturing processes of these containers in the workshop.

Likewise, given the high standard deviation of the average weights of the studied sample (Table 2), the same ranges expressed in *sata* were used to verify whether their measurements corresponded logically to some type of standardization or metrological normalization. Subsequently, it was observed that the sample reflected the existence of groups of amphorae weighing around 3 talents 15 minas, 2 talents 45 minas, 2 talents 30 minas and 2 talents respectively.

Therefore, we were able to hypothesize that the undeniable monetary value of the salted fish contained in the amphorae T-11213 justified the preparation of vessels according to different sizes, following a weight standard, marked by the measurement of the Ugaritic talent. If the Ugaritic pattern was used in Gadir for the manufacture of these amphorae and for the minting of its coins, it is still necessary to go a step further and launch hypotheses that relate the content and its monetary value, and try to formulate approximations to the prices that these products may have reached in Antiquity. Following this same methodology, it will also be interesting to verify if these same weight patterns were used for the manufacture of other objects and to measure the value of other tradable products. Thus, this method and data could lead to the development of many interesting lines of research for future consideration.

7. REFLECTIONS AND IMPROVEMENT OF THE METHOD

The method of quantification proposed in these pages is easily applicable, thanks to the digital tools which are universally available today, to all types of ceramic containers of any chronological and cultural environment. Thus, its utility is based both on its relative ease and speed and on the enormous possibilities for its use in diverse lines of investigation. The error in the volumetric quantification attributable to this methodology is based primarily on the collection of data in the laboratory. This is an extremely important factor that must be taken into account when carrying out a protocol of study for ceramic items exceeding purely typological interests and involving the analysis of quantitative factors, essential for the study of ancient economic history.

On the other hand, it can be argued that the limitation of the method lies in the quality of the data from the archaeological record, which mostly offers fragmented and partial objects. By including vessels with reconstructed profiles, it must be admitted that the method utilizes estimative data. Furthermore, the data would only be totally accurate when the specimens are complete, a circumstance which is usually quite anomalous in Archaeology. Therefore, it is important to emphasize again the significance of the meticulous treatment of the complete specimens that have been recovered and the detailed collection of data on their weight, dimensions, etc. at the laboratory. Obviously, the sample with the greatest number of complete specimens will be the most reliable, and, in addition, the means obtained in the volumetric calculation and in the reconstruction of the spline of the internal profile of each item should serve as a template and as a control method for the digital reconstruction of the fragmented pots.

It is possible to go a step further and argue that, in the case of the T-11213 amphorae, the typological classification proposed so far is insufficient, since all the specimens included in the analyzed sample have been considered *a priori* as equals, belonging to the same typological group. This classification was considered according to profile, length and external and morphological characteristics, without taking into account the capacity. However, the application of this method makes it possible to distinguish at least 4 different groups within this sample of T-11213 according to the results of the calculation of the volume in terms of Ugaritic and Gaditanian metrology. Therefore, the quantification of the volumes contained by any archaeological vessel should be considered key to the compendium of any typological organization. Consequently, the capacity must be taken into account as a factor of importance equal to other details such as the external typological characteristics or the quality of the ceramic.

Furthermore, volume should be considered as the essential differentiating feature of transport vessels at the time of their manufacture and filling, as they were, after all, intended to carry a precious product for trading. For this reason, when establishing ceramic typological categories, it must be taken into account that they would have been manufactured with higher or lower capacities and with the intention of carrying a specific volume according to metrological measures based on standardized values of a concrete pattern.

Therefore, it is important to approximate the measures of the reconstructed containers to values according to the pattern followed by each typological series and reconstruct the curve of their profile accordingly. The results must then be checked to see if, as a result, they are better approximated to the means of these normalized measures. It could also be noted that, logically, it would have been strange to use different patterns for the manufacture of the same container at the same time since this would have resulted in a delay in the profit of the sale of the traded product and this loss would have been avoided by all possible means.

In conclusion, it is clear that the reconstruction of incomplete ceramic pot curves must be made according to the longitudinal means of the full vessel, its spline and the volumetric groups obtained after its comparison with the known metrological systems. To do this, it is proposed that one make a spline or profile curve that draws each of the volumes of the metrological measurements of the pattern used to make these containers to obtain a standard pattern to approximate the reconstruction of the profiles of the samples in the laboratories, thus providing more accurate data.

Equally, the great possibilities that the application of this simple mathematical method can offer for the historical and archaeological knowledge of any chronological period and cultural region are clear. Once these data are obtained, conclusions may be reached on economic, commercial, and consumption patterns, as well as labor forces necessary for the manufacture of a given product, the use and dissemination of weight patterns, etc., among many other lines of research hitherto not quantifiable in strictly numerical terms.

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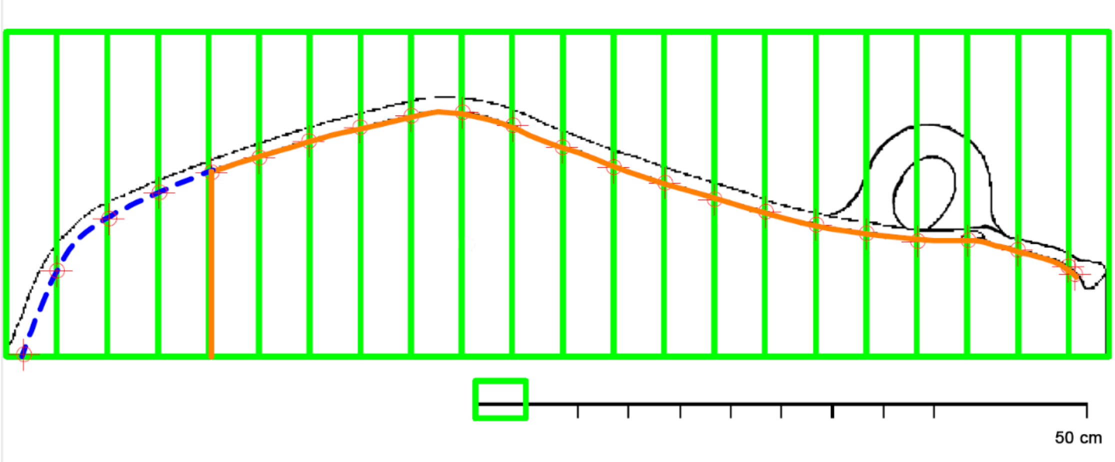


Figure 1. Example of digitizing the profile of the Amphora T-11213 C through *AutoCad*® (Authors' elaboration).

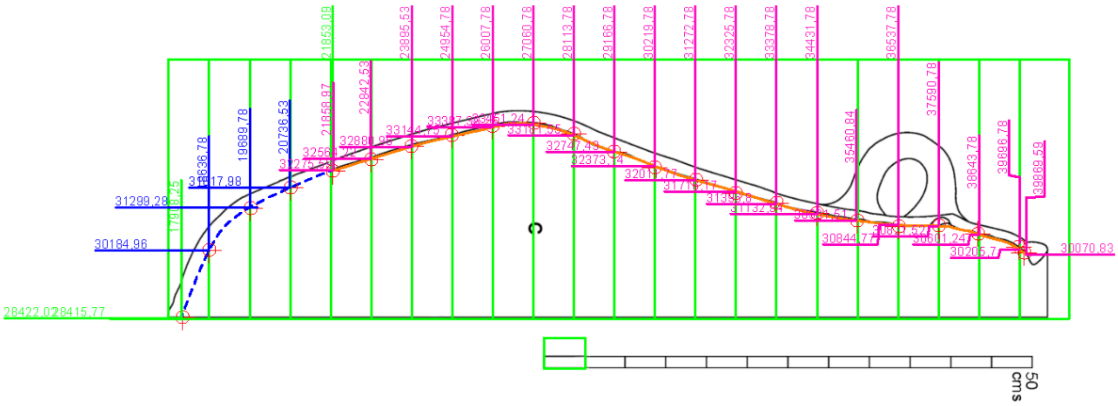
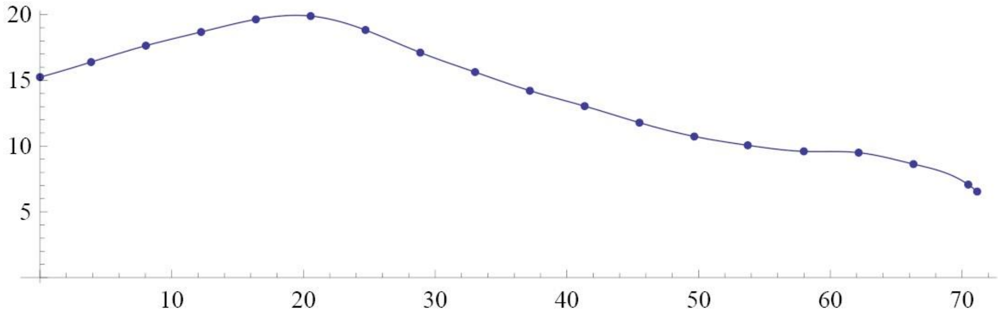


Figure 2. Example of taking coordinates of the regular control points established in the Amphora T-11213 C in *AutoCad*® (Authors' elaboration).

```
data = ( 0.`     15.25`
         3.88`   16.39`
         8.04`   17.64`
         12.23`  18.68`
         16.39`  19.64`
         20.55`  19.89`
         24.71`  18.83`
         28.87`  17.11`
         33.03`  15.63`
         37.19`  14.21`
         41.35`  13.04`
         45.51`  11.78`
         49.67`  10.73`
         53.73`  10.06`
         57.99`  9.6`
         62.15`  9.5`
         66.31`  8.63`
         70.47`  7.07`
         71.15`  6.54` );

sp = Interpolation[datos, Method → "Spline"];
grafpunt = ListPlot[datos, AspectRatio → Automatic];
grafSp = Plot[sp(x), {x, 0.00, 71.15}, AspectRatio → Automatic];
Show[grafpunt, grafSp]
volume = NIntegrate[π sp(x)^2, {x, 0.00, 71.15}]
```

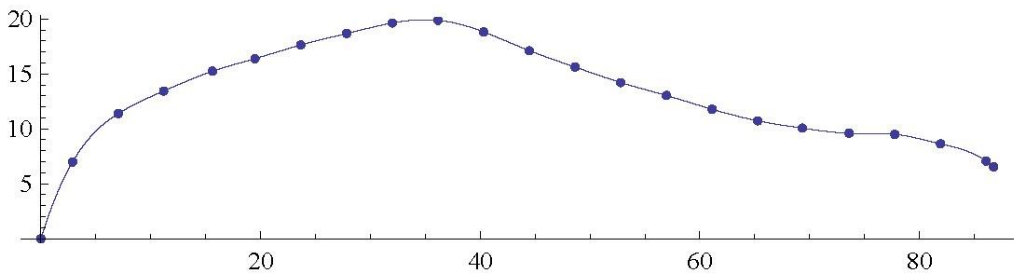


48 696.3

Figure 3. Example of calculation of the volume preserved and reconstructed of the amphora T-11213 C in *Mathematica®* (Authors' elaboration). a. Conserved data results. b. Reconstructed data results.

$$\textbf{data} = \begin{pmatrix} 0.` & 0.` \\ 2.88` & 6.99` \\ 7.04` & 11.39` \\ 11.17` & 13.44` \\ 15.61` & 15.25` \\ 19.49` & 16.39` \\ 23.65` & 17.64` \\ 27.84` & 18.68` \\ 32.` & 19.64` \\ 36.16` & 19.89` \\ 40.32` & 18.83` \\ 44.48` & 17.11` \\ 48.64` & 15.63` \\ 52.8` & 14.21` \\ 56.96` & 13.04` \\ 61.12` & 11.78` \\ 65.28` & 10.73` \\ 69.34` & 10.06` \\ 73.6` & 9.6` \\ 77.76` & 9.5` \\ 81.92` & 8.63` \\ 86.08` & 7.07` \\ 86.76` & 6.54` \end{pmatrix};$$

```
sp = Interpolation[datos, Method → "Spline"];
grafpunt = ListPlot[datos, AspectRatio → Automatic];
grafSp = Plot[sp(x), {x, 0.00, 86.76}, AspectRatio → Automatic];
Show[grafpunt, grafSp]
volume = NIntegrate[π sp(x)^2, {x, 0.00, 86.76}]
```



55 007.9

Figure 3. Example of calculation of the volume preserved and reconstructed of the amphora T-11213 C in *Mathematica*® (Authors' elaboration). b. Reconstructed data results.

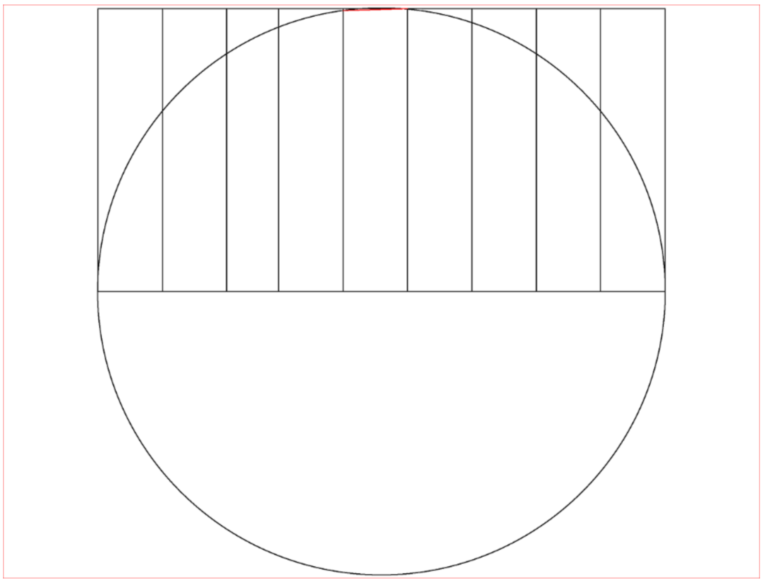
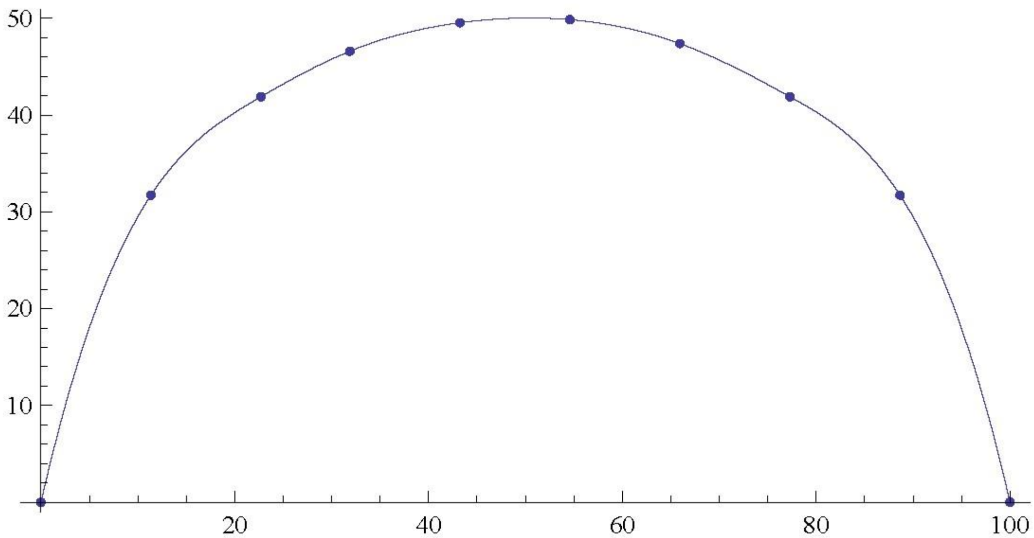


Figure 4 Calibration of the method from the drawing of a semi-circle in *AutoCad®* (Authors' elaboration).

$$\textbf{data} = \begin{pmatrix} 0.` & 0.` \\ 11.35` & 31.73` \\ 22.71` & 41.9` \\ 31.87` & 46.6` \\ 43.23` & 49.54` \\ 54.58` & 49.86` \\ 65.94` & 47.39` \\ 77.29` & 41.9` \\ 88.65` & 31.73` \\ 100.` & 0.` \end{pmatrix};$$

```
sp = Interpolation[datos, Method → "Spline"];
grafpunt = ListPlot[datos, AspectRatio → Automatic];
grafSp = Plot[sp(x), {x, 0., 100}, AspectRatio → Automatic];
Show[grafpunt, grafSp]
volume = NIntegrate[π sp(x)^2, {x, 0., 100.}]
```




```
518 666.
```

Figure 5. Calibration of the method from the calculation of the volume of the proposed hemisphere in *Mathematica*® (Authors' elaboration).

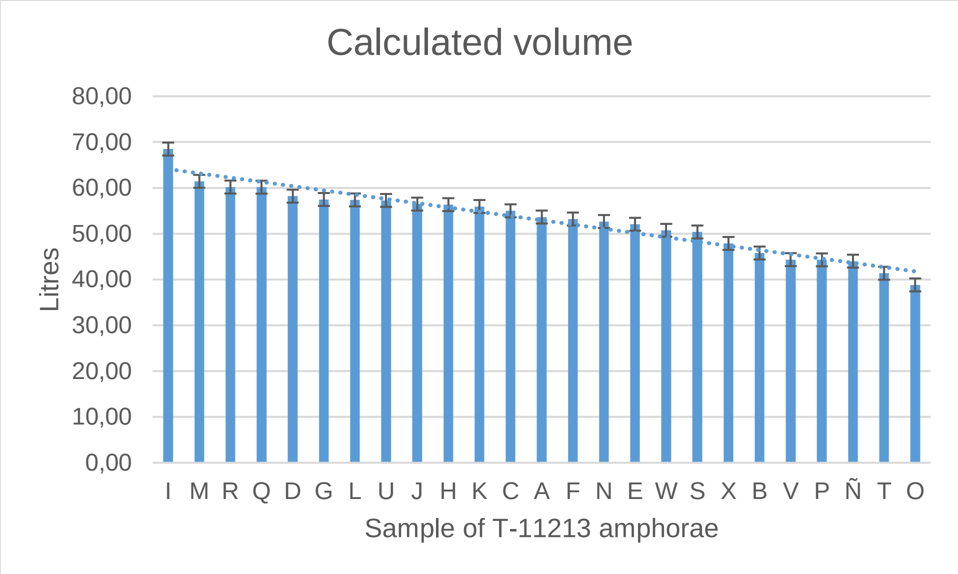


Figure 6. Calculated volumes of the amphorae sample by the described method (Authors' elaboration).

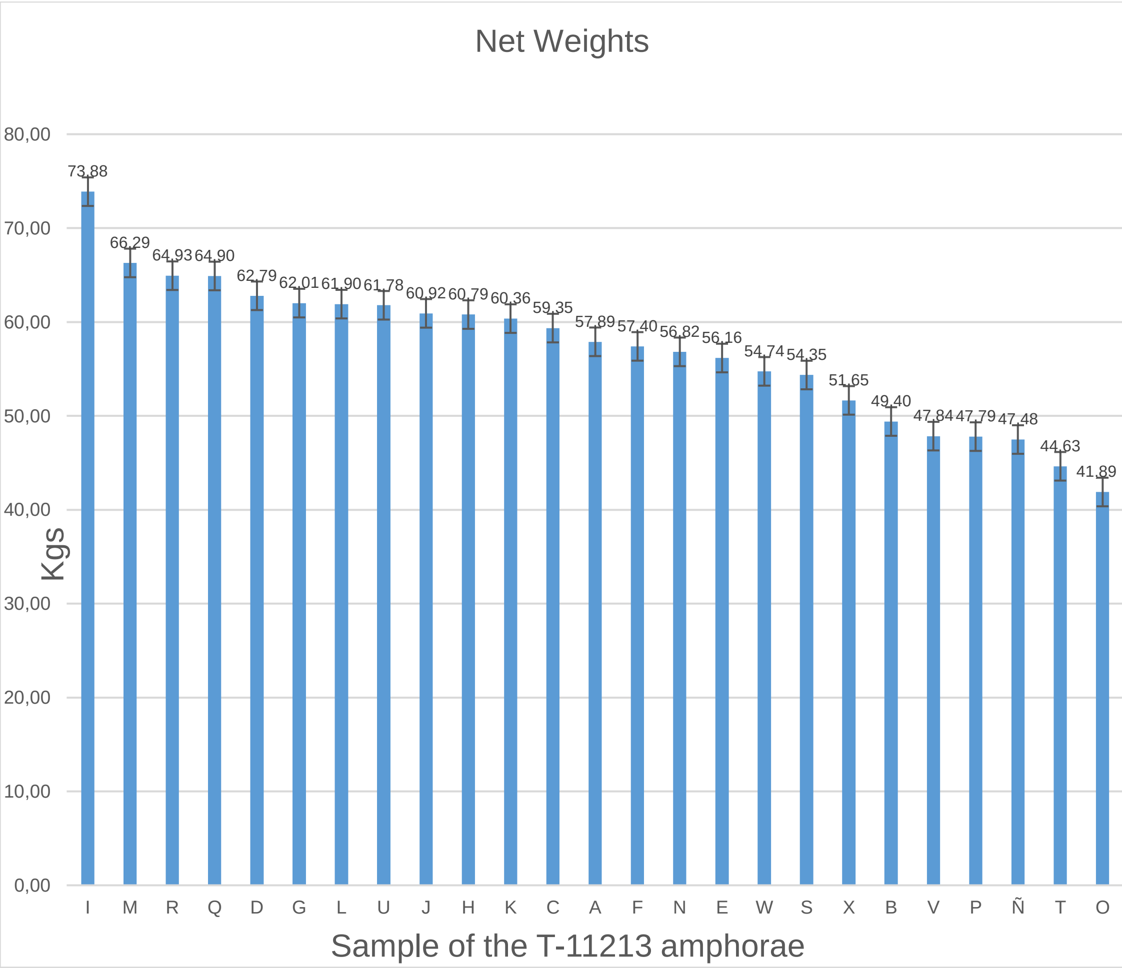


Figure 7. Calculated weights on the amphorae sample by the described method (Authors' elaboration).