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Dimensional analysis of superplastic bulge forming

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Abstract

Free-Bulge Forming (FBF), due to its simplicity, has been widely used as the main featuring test for superplastic forming (SPF) for a large range of materials. Numerous tests have been performed analytically, experimentally and numerically as a way to determine relevant properties of the material and to characterize the whole process. Nevertheless, the high number of variables involved in a SPF process hampers its analysis. This work applies Dimensional Analysis to reduce the number of variables to facilitate the characterization of SPF processes by analyzing the most relevant dimensionless parameters and developing a broad spectrum model of the FBF as a function of them.

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Keywords: Superplastic Bulge Forming; Free-Bulge Forming; Dimensional Analysis

1. Introduction

SPF is a quite specific topic of study in the major field of metal forming techniques. Due to the special conditions of inert atmosphere and work temperature, the equipment is expected to fulfill high mechanical requirements.

The FBF test is probably the most performed essay for SPF analysis, being increasingly applied since the beginning of the use of SPF techniques until now.

During the FBF test, a circular blank sheet is let to expand freely through a circular cavity. The die is designed in order to minimize the contact with the blank within the forming time. The contact can be limited to a perimeter flat surface that ensures the clamping conditions of the sheet and to a curved entry surface that prevents stress concentrations in corners. In the course of the process, the blank takes the form of a circular dome completing the shape as the height of the apex dome reaches the radius of the cavity, see Fig. 1.

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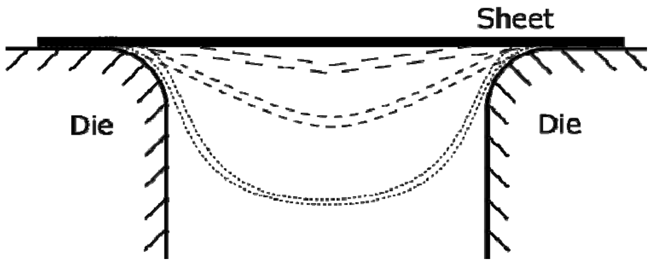

Fig. 1. Basic scheme of the free-bulge forming evolution.

Traditionally, the basic variables for describing the process have been the time evolution of the maximum height and the thickness distribution. Among the first studies, Jovane [1] developed a complete analytical formulation that was also compared with experimental tests. Later, a similar analytical formulation was established by Belk [2]. However, that test was focused into the determination of the mechanical properties of the material as a function of the results of the FBF tests. More analytical studies for FBF can be found in [3-5]. In the late 90s, finite elements models started to be used for the analysis of the basic variables during the forming process [6].

More recently, FBF is still used to characterize the quality of the process and to determine material parameters. The studies using experimental and numerical tests have been applied to several materials such as AA5083 [7], stainless steel [8], Inconel [9], AZ31 [10-13] or Amg6 [14]. However, this diversity of materials, together with the high variety of pressure conditions and geometries makes a global characterization of the process difficult.

Dimensional Analysis [15] has proven to be a powerful analysis technique in many disciplines like fluid mechanics or thermodynamics. Thanks to this technique, different tests can be compared by using the minimum number of variables through dimensionless parameters.

However, the use Dimensional Analysis in mechanics of solids, and more specifically in manufacturing technology, is more unusual. One of these exceptions is [16], in which Shlomchak makes use of a dimensional model to study the rolling of steel sheets.

Consequently, the objective of this work is to develop a complete study based on Dimensional Analysis and apply it to the numerous previous works that use FBF tests. A thorough analysis of the different dimensionless parameters will let us understand their influence in the variables that define the quality of the process, and will serve as a reference point for the developing of a broad spectrum model of FBF tests.

2. Formulation

2.1. Free-Bulge formulation

A complete analytical formulation of the FBF behavior has been developed by Enikeev [5]. The formulation starts setting an axisymmetric geometrical sketch, see Fig. 2. The sketch is defined by a constant die radius, a, a time dependent dome radius, R, the height at the apex-dome, h, and the semi-angle α.

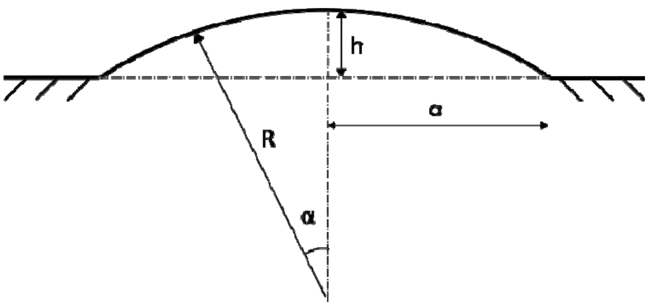

Fig. 2. Geometrical scheme of free-bulge forming.

The dome radius is related to the die radius and the height apex-dome by the relation

$$R = \frac{a^2 + h^2}{2h} \tag{1}$$

Using the die radius to normalize the height apex-dome $\left(H = \frac{h}{a} \right)$ the dome radius can be redefined as

$$R = a\left(\frac{H^2 + 1}{2H} \right) \tag{2}$$

The stretch of an undeformed line, a, to a deformed state, $R\alpha$, can be related as

$$\frac{R\alpha}{a} = \frac{\alpha}{\sin\alpha} \tag{3}$$

This last relationship is forward used to relate the thickness evolution as a function of the semi-angle. Assuming the incompressibility of the process

$$\pi S_o a^2 = \pi S \left(R\alpha \right)^2 \tag{4}$$

where S_o is the initial thickness and S the thickness at the current state. Thus, S can be written as a function of α as

$$S = S_o \left(\frac{\sin\alpha}{\alpha} \right)^2 \tag{5}$$

The formulation is completed with the Norton-Hoff constitutive equation [17], a power-law constitutive equation where an effective stress, σ_p, is related to an effective strain rate, ξ_p.

$$\sigma_p = K \xi_p^m \tag{6}$$

where the effective stress and strain rate can be written in terms of the deviatoric components of the stress and rate of deformation tensors as

$$\sigma_p^2 = \frac{3}{2} \sigma_d : \sigma_d \qquad \xi_p^2 = \frac{2}{3} \xi_d : \xi_d \tag{7}$$

The effective stress is related to the external pressure, p, taking the principal stress at the apex-dome

$$\sigma_p = \frac{pR}{2S} \tag{8}$$

and the effective strain rate can be written as a function of the thickness evolution, $\dot{S}$, as

$$\xi = -\frac{\dot{S}}{S} \tag{9}$$

Finally, the evolution of the dome can be written as an implicit equation of the external pressure as

$$p = \left(\frac{2S_o}{a}\right) K \left(\frac{\sin^3\alpha}{a^2}\right)\left[2\dot{\alpha}\left(\frac{1}{\alpha} - \cot^{-1}\alpha\right)\right]^m = const \tag{10}$$

2.2. Dimensional analysis

The first formulation of the boundary value problem in dimensional analysis applied to SPF was initially proposed by Padmanabhan et al. [17]. His methodology starts settling the necessary both system of equations and boundary conditions to describe the problem, i.e. the continuity equation, the continuum equilibrium equation and the constitutive equation.

Later on, these equations and a set of typical values of length, velocity, stress and acceleration were used to build on a set of dimensionless equations. During the normalization process, a set of five dimensionless parameters arises.

The formulation starts by defining a volume Ω and the boundaries Γ of the body to study. Assuming, as usual, an incompressible behavior of the material, the density ρ remains constant during the process. Thus, the continuity equation in its differential form is:

$$\vec{\nabla} \cdot \vec{v} = 0 \tag{11}$$

where $\vec{v}$ is the velocity vector at each point. Consequently, the continuum equilibrium equation can be formulated in terms of per unit of volume as

$$\rho\left(\frac{d\mathrm{v}_i}{st} - F_i\right) = \frac{\partial \sigma_{ij}}{\partial x_j} \tag{12}$$

where F_i comes from the body forces and it has acceleration units and the σ_{ij} are the Cauchy stress tensor elements. The constitutive equation can be written in its scalar form as

$$\sigma_e = \Phi\left(s, \xi_e\right) \tag{13}$$

where s represents the deformation history and ξ_e is the strain rate. It is well known that the constitutive equation for superplastic behavior of materials is normally written as a power-law relationship between an equivalent stress and an equivalent strain rate. [17]

The boundary value problem is completed with a set of boundary conditions over the surface Γ of the body. The total surface is divided in three different parts depending on the kind of boundary condition applied to it. $\Gamma = \Gamma_k \cup \Gamma_s \cup \Gamma_c$. Γ_k refers to the kinematic boundary conditions, e.g. the clamping condition around the perimeter of the blank.

$$\Gamma_k \subset \Gamma: \ \vec{\mathrm{v}} = \vec{f}\left(x_b, t\right) \tag{14}$$

Γ_s refers to the dynamic boundary conditions, e.g. the pressure applied to both sides of the blank sheet.

$$\Gamma_s \subset \Gamma: \ \sigma_{ij} n_j = q_i \tag{15}$$

and Γ_c refers to the mixed boundary conditions, in which a slide motion between the blank and the die is related to the contact pressure and the friction coefficient.

$$\Gamma_c \subset \Gamma: \ \tau_m = \varphi\left(\sigma_N, \left|\Delta\vec{w}\right|, \mu, \ldots\right) \tag{16}$$

where τ_m, the shear stress applied to the blank by the die, will depend at least on σ_N, as the normal stress, $|\Delta\vec{w}|$, as the relative tangential motion, and μ as the frictional coefficient.

In order to normalize the equations and the boundary conditions, a set of characteristic values are needed: typically a characteristic length l_o, velocity v_o, stress σ_o, acceleration g_o, time t_o and strain rate ξ_s. This leads us to a new system of normalized equations in which five different dimensionless parameters arise. The continuity equation remains the same and do not add any dimensionless parameters,

$$\vec{\nabla} \cdot \vec{v} = 0 \tag{17}$$

where $\vec{v}$ is the normalized variable of velocity. Two dimensionless parameters (κ_1, κ_2) can be obtained when the equilibrium equation is normalized

$$\frac{dv_i}{d\tau} = \left(\frac{\sigma_o}{\rho \mathrm{v}_o^2}\right)\frac{\partial \tau_{ij}}{\partial \lambda_j} + \left(\frac{l_o g_o}{\mathrm{v}_o^2}\right)\eta_i \tag{18}$$

$$\kappa_1 = \frac{\sigma_o}{\rho \mathrm{v}_o^2} \qquad \kappa_2 = \frac{l_o g_o}{\mathrm{v}_o^2} \tag{19}$$

The constitutive equation allows to obtain a third dimensionless parameter

$$\sigma_e = \Phi\left(s, \left[\frac{\mathrm{v}_o}{\xi_s l_o}\right]\xi_e\right) \tag{20}$$

$$\kappa_3 = \frac{\xi_s l_o}{\mathrm{v}_o} \tag{21}$$

The fourth and fifth dimensionless parameters appear from the boundary conditions. The fourth is the frictional coefficient itself

$$\kappa_4 = \mu \tag{22}$$

Meanwhile the last one arises from the dynamic boundary condition

$$\kappa_5 = \frac{q_o}{\sigma_o} \tag{23}$$

3. Methodology

FBF has been usually performed following one of these two strategies:

- Pressure is remained constant during the process
- Pressure is externally controlled in order to reach and maintain a constant strain rate

In this study, a constant pressure has been assumed because it is the most widespread technique as the strain rate in this condition is also approximately constant during most of the forming time. The radius of the die, a, has been chosen as the characteristic length of the FBF test, l_o. The characteristic velocity v_o has been calculated according to l_o and the forming time, t_o, in which the apex dome reaches a height equals to a. Thus, the height and the time were normalized between 0 and 1, where 0 corresponds to the undeformed configuration and 1 corresponds to the final configuration of a circular dome.

Consequently, a height-time (h,) curve was normalized to (H,) where both dimensionless variables are in the range of [0,1]. Once (τ) is obtained, the strain-rate evolution at the apex dome is evaluated analytically as

$$\xi(t) = -\frac{2H}{1+H^2}\dot{H} \tag{24}$$

The characteristic strain-rate ξ_s is obtained directly from the strain-rate evolution as an average of the value within the quasi-constant period. ξ_s is used subsequently in order to evaluate the characteristic stress applying the constitutive equation

$$\sigma_o \sim \xi_s^m \tag{25}$$

The computation of the dimensionless parameters is now straightforward. From the five dimensionless parameters, only three are relevant at first: κ_1, κ_3, κ_5. κ_2 is neglected because it is normally three order of magnitude lower than κ_1, and κ_4 is omitted since it refers to the frictional parameter and its influence will be limited to the clamp surface and the curved entry. Thus, κ_1, κ_3 and κ_5 will be computed as

$$\kappa_1 = \frac{\sigma_o}{\rho \mathrm{v}_o^2} \qquad \kappa_3 = \frac{\xi_s l_o}{\mathrm{v}_o} \qquad \kappa_5 = \frac{q_o}{\sigma_o} \approx \frac{2}{AR} \tag{26}$$

where q_o will take the value of the external pressure p, and AR is the aspect ratio of the blank sheet, being the ratio of the radius and the thickness. This latter expression can be obtained using (8).

4. Results and discussion

In this work, the results of four different studies [3, 5, 18, 19] performing FBF tests in four materials have been analyzed. In these studies, the height position of the apex dome has been measured together with the forming time until the circular dome is finished.

The four studies can be classified according to AR. This geometrical parameter is expected to be essential in the characterization process, being determinant to assess the external pressure which ensures a specific strain rate that let the superplastic behavior. Moreover, the comparison between dimensional equivalent models requires that geometry fulfils homothety.

A summary of the twelve analyzed tests is shown in Table 1, together with the computed dimensionless parameters and the forming time as a quality parameter of the process.

Table 1. Main data of the FBF tests analyzed.

Author (Year)	Material	AR	Pressure (MPa)	κ_1	κ_3	κ_5	t_0 (s)
Song (1986)	ZnAl22	25	0.6	$3.98x10^{10}$	0.533	0.051	160.85
Song (1986)	ZnAl22	25	0.8	$1.18x10^{10}$	0.522	0.055	87.49
Enikeev (1995)	Ti-6Al-4V	35	0.5	$5.63x10^{12}$	0.564	0.036	1500.94
Enikeev (1995)	Ti-6Al-4V	35	0.7	$1.65x10^{12}$	0.588	0.035	678.314
Enikeev (1995)	Ti-6Al-4V	35	1.0	$4.39x10^{11}$	0.595	0.035	291.47
Franchitti (2008)	Az31	35	0.16	$6.01x10^{12}$	0.581	0.032	809.50
Franchitti (2008)	Az31	35	0.29	$6.73x10^{11}$	0.539	0.032	200.34
Estébanez (2011)	PbSn60	100	0.06	$1.02x10^{10}$	0.539	0.012	122.78
Estébanez (2011)	PbSn60	100	0.07	$7.26x10^{9}$	0.545	0.013	99.45
Estébanez (2011)	PbSn60	100	0.08	$4.22x10^{9}$	0.621	0.012	69.35
Estébanez (2011)	PbSn60	100	0.09	$2.33x10^{9}$	0.521	0.013	50.13
Estébanez (2011)	PbSn60	100	0.1	$2.00x10^{9}$	0.538	0.014	45.36

A first examination of the results shows a low variability in the parameter κ_3, with a mean value of 0.556 and a standard deviation of 0.032. However, there is an exponential relation between κ_1 and the forming time, as seen in Fig. 3. κ_5 values are, as expected, only affected by the aspect ratio (26), see Fig. 4.
The validity of the analysis can be appreciated if two different tests with similar dimensionless parameters are selected, as the third in Enikeev (1995) and the second in Franchitti (2008), Table 1. A similar forming time is needed to perform the process, despite the differences in material and external pressure.

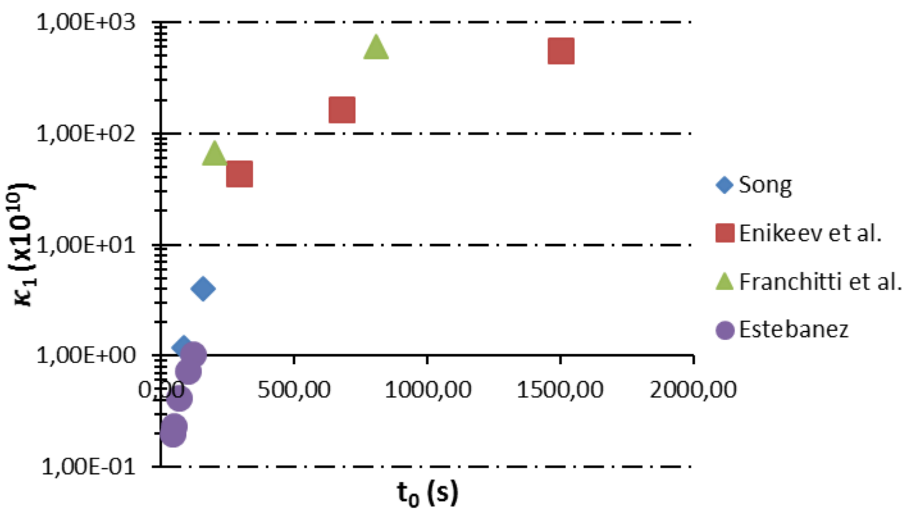

Fig. 3. κ_1 evolution as a function of forming time.

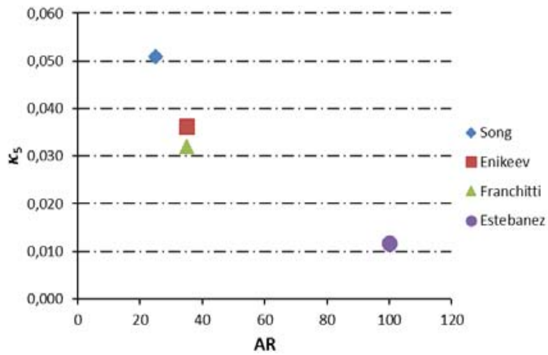

Fig. 4. κ_5 evolution as a function of AR.

5. Conclusions

The first application, to our knowledge, of Dimensional Analysis to Superplastic Forming has been performed. The free-bulge forming test was chosen to characterize the process and to analyse the dimensionless parameters. This choose was based on its simplicity and the high numbers of tests that have been performed in different materials, aspect ratios and pressure conditions. Thus, a set of tests has been analysed to calculate the values of the relevant dimensionless variables in the process. Finally, a comparison between different tests has been performed, offering consistent results which show that dimensional analysis can be a useful tool for describing SPF processes.

We expect that broadening this study to a wider variety of tests will results in a better understanding of how the dimensionless parameters are relevant for the process, as well as to asset the correlations found in this study.

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